Quality Ladders, Competition and Endogenous Growth

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The Conventional View

- innovation along quality ladder, driven by short-term monopoly power
- fixed costs make short-term monopoly power essential to innovation
- fixed cost = “increasing returns to scale”
- innovation is unambiguously good
- rate of innovation limited by diminishing returns: as rate of innovation increases, marginal cost of innovation goes up, limiting equilibrium and efficient rate of increase
- primary theoretical tool to account for the dependence of technological progress on fundamentals such as patience and cost
- Romer [1990], Grossman and Helpman [1991], Aghion and Howitt [1992]

Our Story

- each innovation opens door to growth on a new rung of the quality ladder
- as opportunities opened by an innovation are exhausted becomes both socially and privately optimal to introduce a new innovation
- fixed costs and monopoly power may exist as an empirical matter but play no essential theoretical role
- existing theory: after radio invented everyone moves immediately to inventing television
- our theory: after radio invented everyone spends resources improving and expanding the production of radios – only after the radio widespread, and gains to further improvement and expansion became small do people move on to invent/produce television
- our story is of course the rule not the exception

The Grossman-Helpman Model

d_j consumption (demand) for goods of quality j

ρ subjective interest rate

$\lambda > 1$ constant measuring increase in quality per step up

$c_t = \sum_j \lambda^j d_{jt}$ quality adjusted aggregate consumption

utility of representative consumer

$$U = \int_0^\infty e^{-\rho t} \log[\, c_t \,] dt$$

unit of output (of each quality) requires one unit of labor

first firm to reach step j awarded legal monopoly over that technology

monopoly lasts only until someone gets to rung $j+1$ at which time all firms have access to technology j

same device used by Aghion and Howitt; very convenient for solving the model

labor numeraire so price of output of technology $j+1$ given by the limit pricing formula $p = \lambda$ (everyone competes to produce j selling at cost one)

intensity of R&D for a firm is denoted by $\tilde{\iota}$

probability of next step during dt is $\tilde{\iota} dt$ at cost of $\tilde{\iota} a_I dt$

E defined as steady state flow of consumer spending

wage rate numeraire, price is λ, monopolist's margin is $\lambda - 1$, share of expenditures is margin divided by price

$$(\lambda - 1)/\lambda = 1 - 1/\lambda$$

cost of getting monopoly a_I, so rate of return $(1 - 1/\lambda)E/a_I$

chance ι of losing monopoly, reducing rate of return by same

in steady state consumer expenditure constant so interest rate in expenditure units equal to subjective interest rate.

equate rate of return to subjective interest rate

$$\frac{(1 - 1/\lambda)E}{a_I} - \iota = \rho$$

resource constraint

$$a_I \iota + E/\lambda = 1.$$

solved for steady state research intensity

$$\iota = \frac{(1 - 1/\lambda)}{a_I} - \frac{\rho}{\lambda}$$

solve also for social optimum research intensity

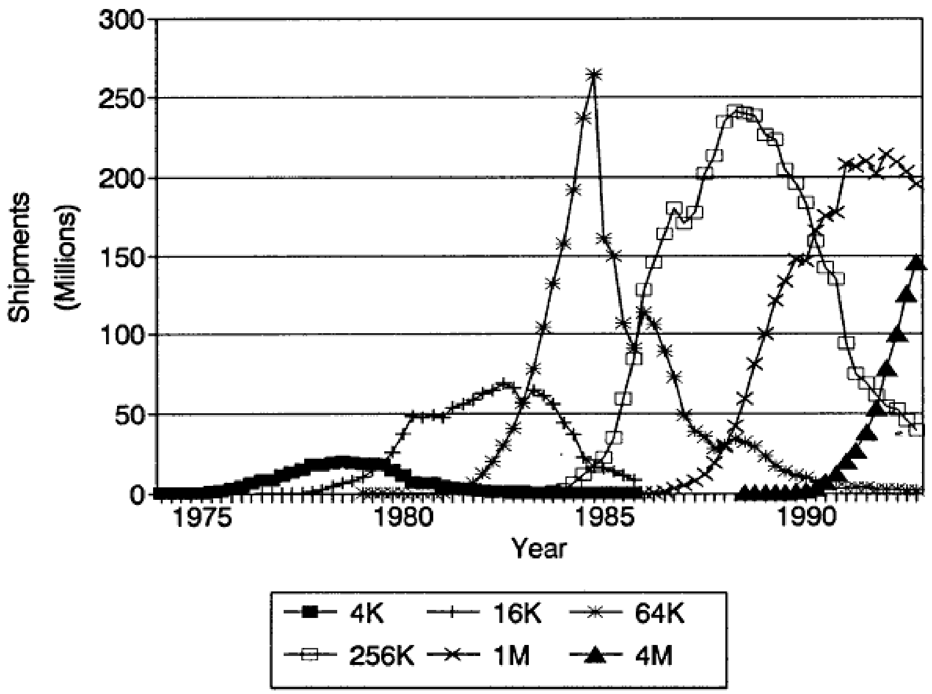
$$\iota * = \frac{1}{a_I} - \frac{\rho}{\log \lambda}.$$

Climbing the Ladder under Competition

profitable to a new good only when the quantity of the old enough to make its price low relative to that of the new one

Irwin and Klenow [1994]: DRAM memory chip, different qualities correspond to capacity of a single chip production of new vintage does not jump up instantaneously, ramps up gradually, new quality introduced when the stock of the old one is large

old vintage phased out gradually as new introduced



price of each vintage falls roughly exponentially

so incentive to introduce the next generation chip keeps increasing

Innovation with Knowledge Capital

same demand structure as Grossman and Helpman

output is produced both from labor and an existing stock of specialized productive capacity

"productive capacity" = capital + knowledge

different rungs correspond to different qualities of capital + knowledge used to produce that output

k_j combined stock of capital and embedded knowledge that goes into producing quality j output

Knowledge Capital

can distinguish between

investment on a given rung – spreading and adopting knowledge of a given type through teaching, learning, imitation, and copying

investment that moves between rungs – innovation or the creation of new knowledge

k_j = *quality* j *knowledge capital*

knowledge capital can have many forms

human knowledge, human capital, books, or factories and machines of a certain design.

Uses of Knowledge Capital

generate more knowledge capital or produce consumption

generate more knowledge capital:

- increase the stock of the same quality of knowledge capital (growth rate $b > \rho$)
- create higher quality (cost of conversion $a > 1$)

or produce output

one unit of quality j knowledge capital + one unit of labor = one unit of quality j consumption

creation of new knowledge costlier than spreading the old $b > \lambda/a$.

h_j flow investment of knowledge capital of quality j in production of knowledge capital of quality $j+1$

motion of quality j stock of knowledge capital is

$$\dot{k}_j + h_j = b(k_j - d_j) + \frac{h_{j-1}}{a}.$$

require $d_j \leq k_j$ and $h_j \geq 0$

allow discrete conversion $\Delta k_{j+1} = -\Delta k_j \,/\, a$

this is an ordinary diminishing return economy: first and second welfare theorems hold; efficient allocations can be decentralized as a competitive equilibrium and vice versa

Pricing of Knowledge Capital

current utility is numeraire, that is, current price of consumption is marginal utility, specifically $1/c_t$

q_{jt} time t price of quality j knowledge capital zero profits on innovation $q_{j+1,t} - aq_{jt} = 0$

rate of return on creation of more knowledge capital of the same quality must equal the subjective interest rate

rate of return is growth of capital plus capital gains

$b + \dot{q}_{jt} / q_{jt} = \rho$ (price falls at rate $b - \rho$)

notice that there is a first mover advantage, because the competitive price of knowledge capital is falling over time

Timing:

- initial unemployment phase (see paper)
- full employment alternation between
- build-up phase
- growth phase

Consumption Value of Knowledge Capital: Full Employment

two different ways to use a small amount ε of quality j knowledge capital over some short time period τ

produce more consumption or more knowledge capital

when we move knowledge capital into the consumption sector must displace existing knowledge capital to free up the labor needed to work with the newly added knowledge capital

move quality j knowledge capital into consumption sector free displaced j' knowledge capital; to do computation suffices to assume diplaced j' is converted immediately back to j

net ε units of quality j knowledge capital displace inferior quality j' in production of consumption then quantity of consumption increased

$$\frac{\lambda^{j} - \lambda^{j'}}{1 - 1/a^{j-j'}} \varepsilon\tau$$

net ε units of quality j knowledge capital displace inferior quality j' in production of consumption then quantity of consumption increased

$$\frac{\lambda^j - \lambda^{j'}}{1 - 1/a^{j-j'}} \varepsilon\tau$$

j knowledge capital used to produce more knowledge capital of same quality get $b\varepsilon\tau$ new units

so: one unit of j knowledge capital perfect substitute for

$$\frac{\lambda^j - \lambda^{j'}}{b(1 - 1/a^{j-j'})}$$

define

$$q_{jt}^{j'} = \frac{\lambda^j - \lambda^{j'}}{b(1 - 1/a^{j-j'})} \frac{1}{c_t}.$$

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$$q_{jt}^{j'} = \frac{\lambda^j - \lambda^{j'}}{b(1 - 1/a^{j-j'})} \frac{1}{c_t}.$$

price q_{jt} of quality j knowledge capital cannot be lower since cannot be strictly profitable to buy knowledge capital and shift it into the production of consumption

$$q_{jt} \geq q_{jt}^{j'},$$

with equality if knowledge capital is used to produce consumption

implication 1: only two qualities of knowledge capital used to produce consumption, and they are adjacent

implication 2: when two qualities of knowledge capital are used to produce consumption, consumption grows at $b - \rho$

The Growth Cycle: The Growth Phase

two adjacent qualities of knowledge capital $j-1, j$ used to produce consumption

consumption grows at $b-\rho$

lasts τ^g defined by initial consumption in efficiency units of λ^{j-1} (all labor used with one unit of $j-1$) and final consumption in efficiency units of λ^j (all labor used with one unit of j)

$$\lambda^{j-1} e^{(b-\rho)\tau^g} = \lambda^j$$

The Growth Cycle: The Buildup Phase

end of the growth phase price of quality $j+1$ knowledge capital is

$$q_{j+1,t} = aq_{jt} = aq_{jt}^{j-1} = a\frac{\lambda^j - \lambda^{j-1}}{b(1-1/a)}\frac{1}{\lambda^j} = \frac{\lambda - 1}{b(1-1/a)}\frac{a}{\lambda}.$$

consumption value of quality $j+1$ knowledge capital is

$$q_{j+1,t}^{j} = \frac{\lambda^{j+1} - \lambda^j}{b(1-1/a)}\frac{1}{\lambda^j} = \frac{\lambda - 1}{b(1-1/a)}.$$

so don't want to use $j+1$ to produce consumption during buildup

consumption remains fixed at λ^j

price of quality $j+1$ capital falls by a factor of λ/a

falls at the constant rate $b-\rho$ so length of phase

$$\tau^b = \frac{\log a - \log \lambda}{b-\rho}.$$

intensity of innovation is rate at which we move up ladder

inverse of the length of the cycle, that is of the sum $\tau^g + \tau^b$ of the two parts, so

$$j^* = \frac{b - \rho}{\log a},$$

Comparison of the Models

Grossman-Helpman model

$$\iota = \frac{(1 - 1/\lambda)}{a_I} - \frac{\rho}{\lambda}.$$

Grossman-Helpman efficient solution

(may correspond better to real institutions than their particular model of monopolistic competition)

$$\iota^* = \frac{1}{a_I} - \frac{\rho}{\log \lambda}.$$

competitive knowledge-capital accumulation

$$j^* = \frac{b - \rho}{\log a}.$$

all models is contrived to get a closed form solution all models give innovation rate as a similar function of cost of innovating and degree of impatience

more patience: intensity of innovation goes up

more costly to innovation: intensity of innovation goes down

minor differences in functional forms – but all based on very special assumption, so not particular significance should be attached to this *substantive differences?*

λ height of ladder run

competitive: neutral – two offsetting effects:

increased intensity of innovation during build-up phase of the cycle – also present in Grossman-Helpman

decreased intensity of innovation during during growth phase

neutrality due to special assumptions; with more sophisticated model could go either way

competitive innovation model has extra widening parameter b, rate at which productive capacity increases

easier to reproduce knowledge capital = larger b is larger: intensity of innovation increases

Grossman-Helpman effectively sets $b = \infty$

competitive knowledge-capital accumulation

output should grow in bursts (the growth phase) punctuated byflat period (the build-up phase)

build-up phase ends when a new vintage of output is produced for the first time

DRAM data from Irwin and Klenow [1994] model

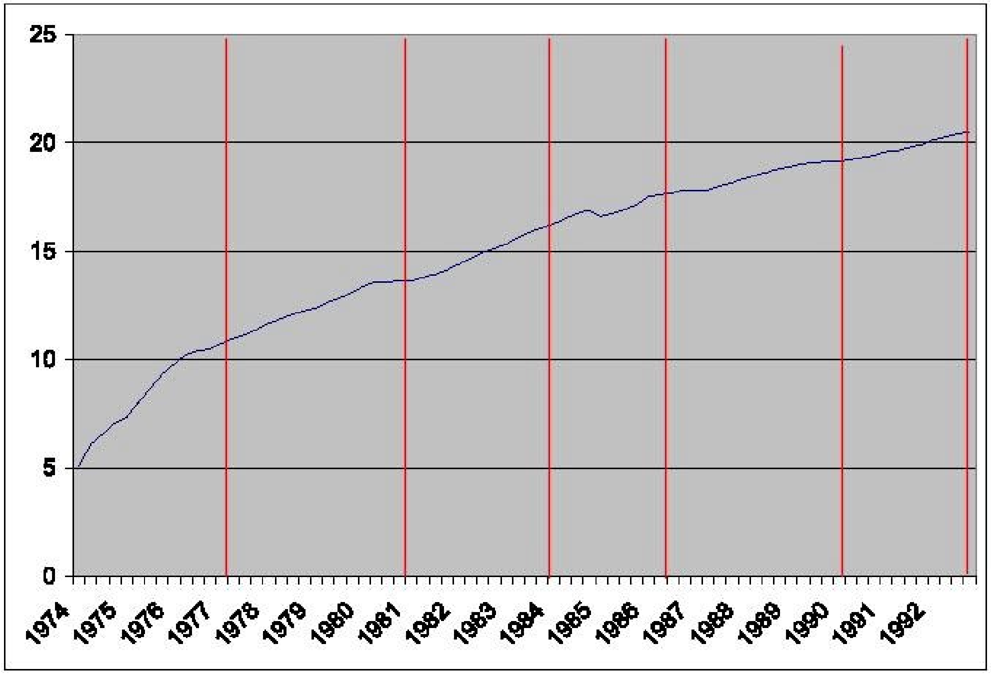
operational definition of “produced for the first time” 5% of the total market for memory

aggregate output in bytes

not every slowdown is followed by a switchover

not every switchover preceded by a slowdown (the 1984 switchover clearly is not)

yet in general, there are period of growth alternating with slowdowns of essentially zero growth associated with the switchovers



buildup phases very short suggesting that λ is not much smaller than a.

Fixed Cost of Knowledge Capital

empirical fact: there is a fixed cost of creating new knowledge

two left-halves of blueprint not a good substitute for left and right half

also fixed costs in producing just about everything: doesn't prevent divisible model from being a useful tool

robustness of model of competitive innovation to fixed costs?

to produce for the first time quality $j+1$ knowledge capital from quality j knowledge capital requires a fixed cost of F units of quality j knowledge capital

results in the creation of $\overline{k} < F$ initial units of quality $j+1$ knowledge capital

for simplicity and notational convenience after fixed cost incurred can convert additional units of quality j knowledge capital to quality $j+1$ knowledge capital at same rate $F/\overline{k} = a$

assume initial quantity of knowledge capital (the single blueprint) does not "flood the market" for knowledge capital

$$\overline{k} \leq k^* \equiv \frac{a^{\frac{b}{b-\rho}} - 1}{a^{\frac{b}{b-\rho}} - a} > 1.$$

(otherwise in Grossman-Helpman case)

take as parameters F, a

in perfectly divisible model exact time at which quality j knowledge capital is converted to quality $j+1$ knowledge capital is a matter of indifference (as long as it is "soon enough)

among divisible equilibria, there is one at which knowledge capital of quality j not converted to knowledge capital of quality $j+1$ until the first moment at which quality $j+1$ knowledge capital is used for the first time in the production of consumption (at the end of build-up and beginning of growth)

unique such equilibrium in which all quality j knowledge capital not needed to produce consumption is converted to quality $j+1$

let $F*$ denote unique amount of quality j knowledge capital not being used in the production of consumption at the end of build-up, and let t be the time at which that build up ends. Then $F^* = k_{jt} - 1$

since we are in a steady state, we can compute

$$F^* = \frac{a^{\frac{\rho}{b-\rho}}}{a^{\frac{\rho}{b-\rho}} - 1} \frac{\lambda^{\frac{\rho}{b-\rho}} - 1}{\lambda^{\frac{\rho}{b-\rho}}} \frac{a-1}{\lambda - 1} \frac{b-\rho}{\rho}.$$

Small Fixed Cost

$$F \leq F*$$

the constraint didn't bind…competition free to work its magic

Large Fixed Cost

$$F > F^*$$

divisible equilibrium not feasible

innovation is not possible at the time at which build-up would usually end, t^b

as time continues to pass and consumption remains constant, capital will continue to grow, so there is a later time at which it will be possible to pay the fixed cost

suppose it happens at $t > t^b$

not a competitive equilibrium: in divisible case, can introduce a small amount of quality $j+1$ knowledge capital at an intermediate time $t^b < t' < t$ and earn a profit

but can't do this with fixed cost

drop from definition of competitive equilibrium requirement that an "early" innovation at a time $t^b < t' < t$ not generate profit at existing equilibrium prices

atomistic equilibrium: individual competitors too small to introduce an innovation on their own, so cannot take advantage of a profit opportunity from innovating, even if one exists

not surprising: many atomistic equilibria

surprising: they are all pretty similar

analyze the zero-profit condition on innovation

"end of growth" at time t – single unit of quality j knowledge capital is used to produce consumption, while a moment earlier both qualities $j-1$ and j were used

price of knowledge capital

$$q_{jt} = \frac{\lambda^j - \lambda^{j-1}}{b(1-1/a)} \frac{1}{c_t} = \frac{\lambda^j - \lambda^{j-1}}{b(1-1/a)} \frac{1}{\lambda^j}.$$

During build up price of quality j capital falls

$$q_{j,t+\tau} = \frac{\lambda^j - \lambda^{j-1}}{b(1-1/a)} \frac{1}{\lambda^j} e^{-(b-\rho)\tau}.$$

quality $j+1$ is used to produce consumption for the first time

$$q_{j+1,t+\tau} = \frac{\lambda^{j+1} - \lambda^j}{b(1-1/a)} \frac{1}{c_{t+\tau}} = a q_{j,t+\tau}.$$

zero profit in innovation.

$$c_{t+\tau} = \frac{\lambda^{j+1}}{a} e^{(b-\rho)\tau}.$$

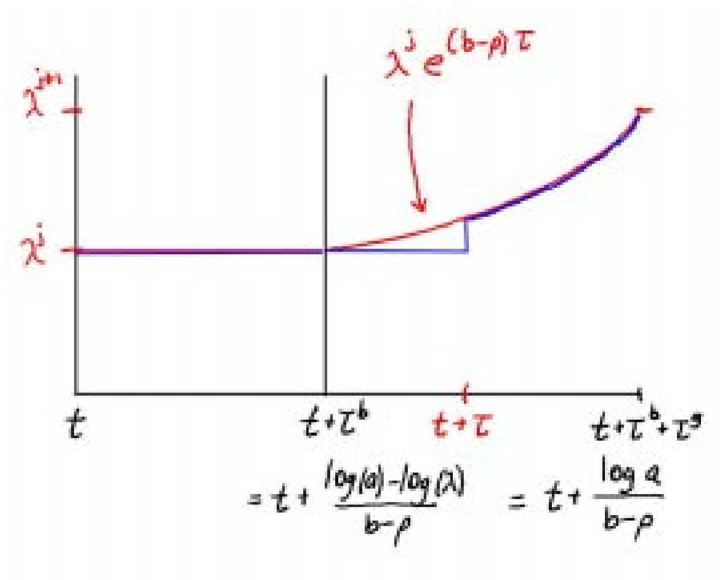
also have $c_{t+\tau} \geq \lambda^j = c_t$, since using higher quality knowledge capital to displace a lower quality must necessarily increase amount of consumption produced divisible competitive equilibrium case must hold with exact equality, so $\tau = \log(a/\lambda)/(b-\rho)$. If not can show profit from introducing innovating a small amount of quality $j+1$ knowledge capital a moment earlier

but we dropped that requirement

can innovation $\tau > \log(a/\lambda)/(b-\rho)$ with a discrete jump in consumption from λ^j to

$$c_{t+\tau} = \frac{\lambda^{j+1}}{a} e^{(b-\rho)\tau}$$

same level it would have been at had innovation taken place earlier and grows at the same rate all of these different paths share the same combined length of build-up and growth, and the same innovation intensity $j^* = (b - \rho)/\log(a)$



feasibility given fixed cost

consumption jump $1 < \xi < \lambda$ so $c_{t+\tau} = \xi c_t = \xi \lambda^j$

determine the amount of quality j knowledge capital required to produce consumption from the equation

$$\lambda^j d_{j,t+\tau} + \lambda^{j+1}(1 - d_{j,t+\tau}) = \xi \lambda^j$$

use steady state condition to determine amount of knowledge capital $F*(\xi)$ that can be converted from quality j to $j+1$

$$F*(\xi) =$$

$$\frac{-1 - \frac{1}{\lambda - 1}\left(a^{\frac{b}{b-\rho}} - \lambda a^{\frac{\rho}{b-\rho}}\right) - \frac{a-1}{\rho}\frac{a^{\frac{\rho}{b-\rho}}}{\lambda - 1}\left((b-\rho)\lambda\left(\frac{\xi}{\lambda}\right)^{\frac{b}{b-\rho}} - b\xi\right)}{a^{\frac{\rho}{b-\rho}} - 1} - \frac{\lambda - \xi}{\lambda - 1}.$$

can be shown to be increasing in ξ

feasibility: $F \leq F*(\xi)$

amount of capital that can be converted at least as great as amount that must be converted due to fixed cost

$F*(\xi)$ increasing in ξ, larger jumps mean can sustain larger fixed cost

under our assumption on $\overline{k}$ always some value such that $F \leq F(\xi)$

to summarize: steady state atomistic equilibria with continued innovation and innovation intensity $j* = (b - \rho)/\log(a)$ exist

entrepreneurial equilibrium?

other robustness