Factor Saving Innovation

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Introduction

endogeneity of aggregate technological progress

we introduce concave model of innovation with three properties concerning technological innovations:

(a) factor saving

(b) implementable only in discrete lumps

(c) endogenous, depending on people's decisions

such circumstances growth can be

(a) path dependent

(b) uneven over time

how the model works

♦ labor saving technological improvement in concave framework

♦ capital either reproduce or produce a “better” kind of capital

♦ better capital requires less **labor** input

♦ low cost technological improvement: “exogenous growth” –

economy grows at fixed rate determined solely by technology,

♦ high cost technological improvement –

growth rate determined by preferences as well

Endogeneity of Growth and Technological Change

- growth due to the accumulation of factors versus growth in productivity of factors – technological advance
- growth rate or the rate of technological advance *endogenous* if depend on subjective discount factor
- in Solow growth model neither growth rate nor rate of technological advance are endogenous
- In Rebelo's [1991] AK model growth rate is endogenous, but the rate of technological advance is not
- Increasing returns such as Lucas [1988] or Romer [1990] both are endogenous

Related Issue:

Romer [1994]

- "technical advance comes from things that people do" not merely "a function of elapsed calendar time"
- argues against concave models of "exogenous" technological change
- endogeneity means that technological innovations should come from "things people do"

Concave Model of New Products

- stylized concave model with many different qualities of capital
- higher levels of total factor productivity naturally associated with higher qualities ofcapital
- fixed and potentially binding labor (or natural resource) constraint
- better quality of capital is labor-saving

investment provokes

- *capital widening*, meaning the total stock of capital grows larger
- *capital deepening* meaning that the quality of the capital stock improves
- because of fixed labor supply, capital deepening necessary for capital widening

technical advances clearly come from things that people do

- ♦ contrary to models where externalities carry the day technological improvements here come from things that people consciously choose to do
- ♦ introduce new technologies when needed to relax labor constraint
- ♦ do not introduce new technologies when such need is absent

In the endogenous case:

- ♦ process of growth is necessarily uneven
- ♦ exhibits a natural cycle with periods of “growth recession”
- ♦ path and innovations exhibit dependence upon initial conditions

The Model

Consumers

infinite economy horizon $t = 1, 2, \ldots$

continuum of homogeneous consumers

consumers value consumption $c_t \in \Re_+$

period utility function $u(c_t)$ bounded below, continuously differentiable, strictly increasing, and strictly concave, satisfies the Inada conditions $\lim_{c \to 0} u'(c) = \infty$, $\lim_{c \to \infty} u'(c) = 0$

lifetime utility $U(c) = \sum_{t=0}^{\infty} \delta^t u(c_t)$; $0 \leq \delta < 1$ common subjective discount factor

$\mu = \sup\{\tilde{\mu} \mid \sum_{t=0}^{\infty} \delta^t u(\tilde{\mu}^t) < \infty\}$ note that $\mu \geq 1/\delta$

Production

consumption produced by activities using labor and capital as inputs

capital is produced from capital, and labor reproduces itself

capital comes in an infinite sequence of different qualities $i = 0, 1, \ldots$

$z = (\kappa, \ell)$ where κ an infinite vector of different quality capital and ℓ a scalar denoting labor

period input space $Z \subset \ell^{\infty}_{+}$ of sequences $(z_1, z_2 \ldots, z_n, \ldots) \geq 0$ with $z_n = 0$ for all but finitely many n

χ_i vector of one unit of capital of quality i

activities $a \in \boldsymbol{A}$ $a = [z(a); z^{+}(a); c(a)]$

input in period t; output of consumption in period t; output of capital in period $t + 1$

a sequence of activities for producing consumption $[\chi_i, 1/\gamma^i; 0, 1/\gamma^i; 1]$

- ♦ for a unit of consumption a unit of capital
- ♦ labor requirement diminishes with quality of capital
- ♦ labor reproduces itself

a sequence of activities for reproducing capital $[\chi_i, 0; \beta\chi_i, 0; 0]$

a sequence of activities for improving capital $[\chi_i, 0; \rho\chi_{i+1}, 0; 0]$

$\beta > \rho$, $\mu > \min\{\beta, \gamma\}$

labor reproduces itself $[0, 1; 0, 1; 0]$

free disposal

endowment κ_0^0 units of quality zero capital and one unit of labor

Equilibrium

$\lambda \in \times_{t=0}^{\infty} \Re_{+}^{\mathcal{A}}$ a production plan, $c \in \times_{t=0}^{\infty} \Re_{+}$ a consumption plan

Definition 1: λ, c are a *feasible allocation* for the initial condition z_0 if

$$1 \geq \sum_{a \in \mathcal{A}} \lambda_0(a) \ell(a)$$

$$\kappa_0^0 \chi_0 \geq \sum_{a \in \mathcal{A}} \lambda_0(a) \kappa(a)$$

$$\sum_{a \in \mathcal{A}} \lambda_t(a) z^{+}(a) \geq \sum_{a \in \mathcal{A}} \lambda_{t+1}(a) z(a)$$

Definition 2: λ^*, c^* solve the *social planner problem* for initial condition z_0 if it solves $\max_{\lambda, c} U(c)$ subject to social feasibility in a feasible production plan $\lambda_t(a) = 0$ if a uses as input any quality of capital greater than t; we call the set of such activities *viable* and denote them by A_t

q_t^i price of quality i capital delivered at time t

q_t^ℓ price of labor delivered at time t

q_t vector of input prices

p_t price of consumption delivered at t

infinite sequence of prices (q, p)

prices q, p and a feasible allocation λ, c are a *competitive equilibrium* if c maximizes $U(c)$ subject to the budget constraint

$$\sum_{t=0}^{\infty} p_t c_t \leq q_0^0 \kappa_0 + q_0^\ell$$

and activities satisfy the *zero profit condition*

$q_{t+1} z^+(a) + p_t c(a) - q_t z(a) \leq 0$ for all $a \in A_t, t = 0,1,\ldots$

with equality if $\lambda_t(a) > 0$

Welfare and Existence

Welfare Theorems: Suppose that λ^*, c^* is a feasible allocation for the initial condition κ_0. Then λ^*, c^* solves the social planner problem if and only if we can find prices q, p such that q, p, λ^*, c^* are a competitive equilibrium.

Existence Theorem: For given κ_0, a competitive equilibrium exists, and there is a unique competitive equilibrium consumption c^*.

Capital Requirements Function

$c_t \leq 1$ set $\eta(c_t) = 0$; $\gamma^{i-1} < c_t \leq \gamma^i$ set $\eta(c_t) = i$

$$\kappa_0^0(c_t) = \begin{cases} \beta^{-t} c_t & \eta(c_t) = 0 \\ \beta^{-t} \left(\frac{\beta}{\rho}\right)^{\eta(c_t)} \frac{\gamma c_t - \gamma^{\eta(c_t)}}{\gamma - 1} + \beta^{-t} \left(\frac{\beta}{\rho}\right)^{\eta(c_t)-1} \frac{\gamma^{\eta(c_t)} - c_t}{\gamma - 1} & \eta(c_t) > 0 \end{cases}$$

amount of initial capital required to produced c_t when it is produced using only qualities $\eta(c_t), \eta(c_t) - 1$

initial capital requirement to produce c

$$\kappa_0^0(c) = \sum_{t=0}^{\infty} \kappa_0^0(c_t)$$

Consumption Correspondence

Define constants

$$\zeta_0 = 1$$

$$\zeta_i = \left(\frac{\beta}{\rho}\right)^{i-1} \frac{\beta\gamma / \rho - 1}{\gamma - 1}$$

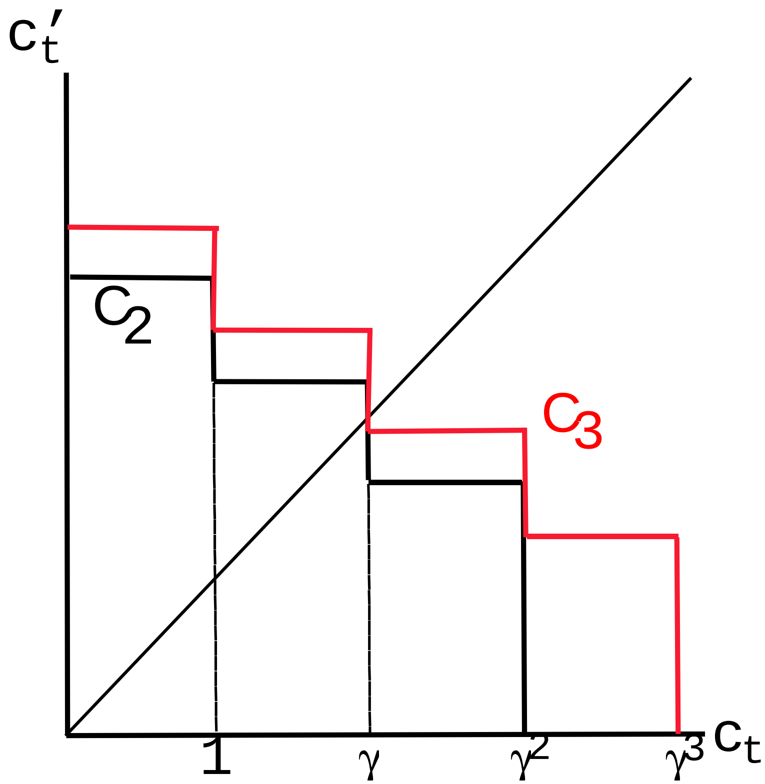
correspondence $c_t' \in C_t(c_t, q_0^0)$ by

$$\begin{array}{ll} u'(c_t') = (\beta\delta)^{-t} q_0^0 \zeta_{\eta(c_t)} & c_t < \gamma^{\eta(c_t)}, \eta(c_t) \le t \\ (\beta\delta)^{-t} q_0^0 \zeta_{\eta(c_t)} \le u'(c_t') \le (\beta\delta)^{-t} q_0^0 \zeta_{\eta(c_t)+1} & c_t = \gamma^{\eta(c_t)}, \eta(c_t) < t \\ (\beta\delta)^{-t} q_0^0 \zeta_{\eta(c_t)} \le u'(c_t') & c_t = \gamma^{\eta(c_t)}, \eta(c_t) = t \end{array}$$

horizontal and vertical line segments forming the steps of "descending" stair

upper-hemi-continuous, convex valued, non-increasing

for given q_0^0 and t exactly one fixed point $c_t{}^* \in (0, \gamma^t]$



Theorem: For given κ_0 the feasible consumption plan $c*$ is an optimum if and only if there exists a q_0^0

$\kappa_0^0 \geq \kappa_0^0(c*)$ with equality unless $q_0^0 = 0$

$c_t* \in C_t(c_t*, q_0^0)$

Moreover, equilibrium prices are given by the following

$$q_t^i = \beta^{-t} \left(\frac{\beta}{\rho} \right)^i q_0^0$$

$$w_t = \gamma^{\eta(c_t*)} \left[\delta^t u'(c_t*) - \beta^t \, (\beta / \rho)^{\eta_t} \, q_0^0 \right]$$

$$q_T^\ell = \sum\nolimits_{t=T}^{\infty} w_t$$

equilibrium production plan is any feasible plan producing c_t* using only quality $\eta(c_t*), \eta(c_t*) - 1$ quality capital

full employment whenever $\eta(c_t*) > 0$

Solow, Growth Cycles and Stagnation

long-run behavior of the economy

three possible outcomes

- *Solow* growth path - new technology introduced every period and the economy grows at the rate γ; provides highest attainable level of consumption every period
- *Stagnation* – only worst technology used to produce consumption and consumption either declines or remains the same over time
- *Growth cycle* - two different qualities of capital used for a period of time, then lower quality capital dropped and a new quality of capital is introduced and so on

The Solow Balanced Growth Path

economy can grow only by moving to more advanced qualities of capital making it possible to increase output from existing labor

when innovation occurs ρ units of new capital are produced for each unit of old capital invested generating an additional demand of $\rho / \gamma - 1$ for labor

$\rho > \gamma$ labor demand is increased

can shift the entire stock of capital from one quality to the next without unemployment

in this case the optimum is to use a new quality of capital each period

rate of technological progress independent of preferences

consumption grows at fixed rate γ

initial capital stock large unique equilibrium is Solow growth path beginning with consuming a unit in period one

if this path is feasible it must be optimal, since it is not possible by any plan to have higher consumption in any period

Theorem: If $\rho > \gamma$ and $\kappa_0^0 \geq \rho/(\rho - \gamma)$ the unique equilibrium is a balanced growth path in which a new technology is introducedevery period, consumption in period t is γ^t, capital also grows at the rate γ and there is full employment in all periods.

prices, factor shares and observable TFP

price of capital is zero if $\kappa_0^0 > \rho/(\rho-\gamma)$, and may be zero even with equality

$\nu_t = 1, \eta_t = t$

consumption prices $p_t = \delta^t u'(\gamma^t)$

Rental rate of labor $w_t = \gamma^t p_t$

real wage $\tilde{w}_t = \gamma^t$, so real wages grow exponentially over time

The Growth Cycle

$\gamma < \rho$ or $\kappa_0^0 < \rho/(\rho - \gamma)$ long run behavior of both consumption and the introduction of new technologies will generally depend upon preferences and in particular on the subjective discount factor δ

two cases: $\delta\beta > 1$ or $\delta\beta \leq 1$

no labor constraint would correspond sustained growth through capital accumulation and stagnation with consumption eventually bounded or decreasing

also true with a labor constraint

General Case

$\delta\beta > 1$

- consumption is non-decreasing
- no upper bound on qualities of capital used to produce consumption

as t increases stairstep correspondence C moves up and to the right

if movement is sufficiently slow ($\delta\beta$ near 1) fixed point will lie on the same segment for several consecutive periods

length of time on a segment determines rate at which new technologies are introduced

system behaves differently on horizontal and vertical segments

horizontal segments – boom - two types of capital used to produce consumption, and consumption grows C shifts upwards

vertical segments – recession - only one type of capital used to produce consumption, and consumption remains constant as C shifts upwards during a recession real wage increases, real price of “higher” quality capital declines until it become profitable to introduce the next higher quality of capital into producing consumption to save on labor

The Continuous Time Limit

time between periods small

CES preferences $u(c_t) = -(1/\theta)[c_t]^{-\theta}$

In addition, take$\delta = e^{-r\Delta}, \beta - e^{b\Delta}$

$\delta\beta > 1$ corresponds to $b > r$

assume that innovations are discrete: extent to which machine i saves on labor relative to machine $i-1$ independent of time

so $\gamma > 1$ independent Δ.

also $\rho = \tilde{\rho}e^{d\Delta} < \gamma, \tilde{\rho} < 1$

calendar time $\tau = t\Delta$

boom

consumption grows at $(b - r)/(\theta + 1)$

$$\tau_b = \frac{\theta + 1}{b - r} \ln \gamma$$

recession

$$\tau_r = -\frac{\ln \tilde{\rho}}{b - r}$$

total cycle length = slower innovation

$$\tau_c = \frac{1}{b-r} \ln\left(\frac{\gamma^{1+\theta}}{\tilde{\rho}}\right)$$

increasing in γ, θ, r decreasing in $b, \tilde{\rho}$

higher quality innovations lead to less innovation, because they make it possible to grow for a longer period of time without hitting the labor constraint

more innovation if the cost of producing capital goes down relative length of the two phases

$$\frac{\tau_b}{\tau_r} = -(1+\theta)\frac{\ln \gamma}{\ln \tilde{\rho}}$$

neither the productivity of the capital widening technology, nor the degree of impatience affect the relative length of booms and recessions

low willingness to substitute consumption over time (high values of θ) have longer (but “less rampant”) booms for a given recession length

improved quality of innovation (high γ) makes it possible to grow for a longer period of time without hitting the labor constraint increasing length of booms, but not recessions

a large cost of innovation is bound to increase the relative amount of time spent in recession average growth rate of consumption over an entire cycle

$$g = \frac{b - r}{1 + \theta - \ln \tilde{\rho} \,/\, \ln \gamma}$$

economies where people are more willing to substitute consumption over time; able to implement more substantial innovations – grow faster

real wage grows during recession, always full employment - countercyclical movement in the labor share of national income

price of a machine of quality i decreases over time relative to consumption and the rate of decrease is uniform across qualities

Stagnation

$\delta\beta \leq 1$

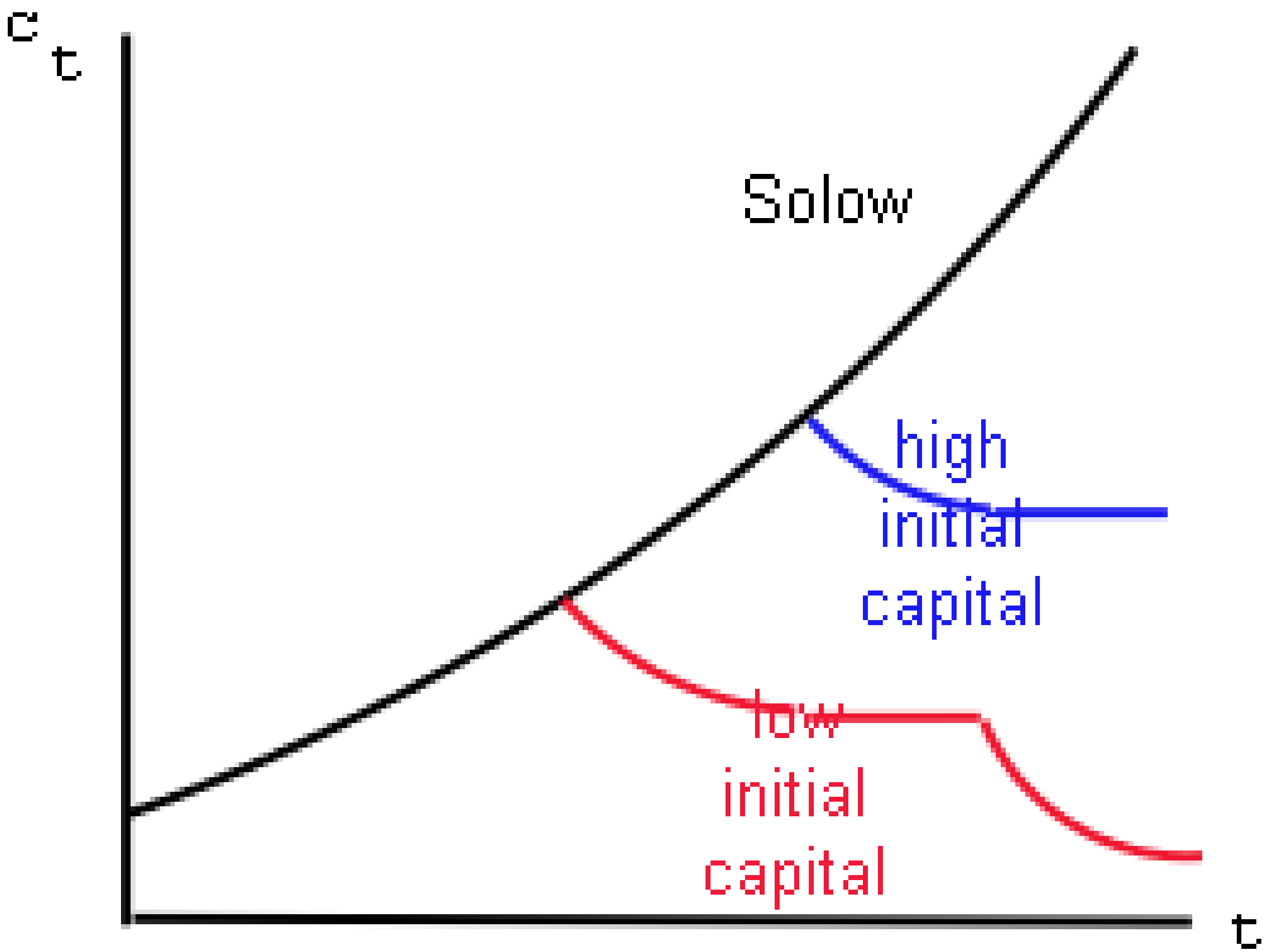
absence of a labor constraint economy remains stagnant with constant consumption if $\delta\beta = 1$ or declining consumption if $\delta\beta < 1$

with labor constraint, if $\rho > \gamma$ and $\kappa_0^0 \geq \rho/(\rho - \gamma)$ equilibrium is Solow path regardless of whether $\delta\beta \leq 1$

introducing labor constraint and possibility of factor saving technological progress changes a stagnant economy into an expanding economy

Theorem: Suppose either $\rho < \gamma$ or $\kappa_0^0 < \rho/(\rho - \gamma)$. If $\delta\beta \leq 1$ there exists a technology I such that no quality of capital greater than I is ever produced, and a period T such that for all $t \geq T$

- If $\delta\beta = 1$, ${c_t}^* = {c_T}^*$
- If $\delta\beta < 1$, ${c_{t+1}}^* < {c_t}^* < 1$
- only worst type of capital used to produce consumption


c
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Solow
high
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Path Dependence

suppose that $\delta\beta < 1$ and $\rho > \gamma$

- if initial capital exceeds $\rho/(\rho-\gamma)$ long run is one of technological innovation and sustained growth
- if initial capital fall a bit short of the threshold in the long-run only the lowest possible quality capital is produced, there is unemployment, and consumption continually falls
- if we compare two economies with different initial capital endowments, one above and one below the threshold, we would discover that they do not “converge”' to the same long-run growth path.

Non Monotonicity

if $\delta\beta < 1$, $\rho > \gamma$, $\beta/(\beta-1) < \kappa_0^0 < \rho/(\rho-\gamma)$ the economy will innovate and grow for some period of time, before falling back into stagnation

rich, but not terribly patient

Conclusion

- an essential input cannot be increased at the same speed as the others
- growth in per capita consumption needs factor saving innovations to take place
- machines that need less of a certain factor than other machines must be more expensive
- factor saving innovations necessarily induce a non-trivial trade-off between capital widening and capital deepening
- consequently rate at which new technologies are introduced becomes endogenous, depending on rate of intertemporal substitution in consumption, on technology and on initial conditions

factor constraint binds in consumption sector only, one fixed factor

- basic message remains the same regardless of such simplifying restrictions
- does not depend upon sector in which the constraint binds
- does not depend upon labor mobility between two sectors
- preliminary versions of perfect labor mobility between sectors gives same qualitative results
- perfect labor mobility not especially more plausible than complete immobility