

# **Growth Cycles and Market Crashes**

Michele Boldrin  
David K. Levine

## ***Introduction***

- Are stock market crashes due to irrationality and are they bad?
- Stylized fact: market rises smoothly, falls abruptly
- Superficially asymmetric information shocks are implausible
- We argue a natural model of technological progress leads to exactly this type of asymmetry
- Based on model of economy growing due to improving technology introduced in Boldrin and Levine [1997]

## *Is There An Asymmetry Between Rises and Falls?*

Standard and Poor's Index annually from 1889-1984  
logarithmic first differences in deviation from mean  
length of runs

Run	+	-
5	2	0
4	2	1
3	2	1
2	9	11
1	9	12
	51	41

## **The Model**

### *The State Space*

$t = 0, 1, 2, \dots$

each  $t$  countable number of states  $\eta_t \in I$

*state history*  $s = (\eta_0, \eta_1, \dots, \eta_t)$  all state histories  $S$

$t(s)$  terminal time,  $\eta_t$  terminal state;  $\tilde{s} \geq s$ ;  $s-1$

probabilities  $\pi_s$  so that  $\sum_{s:t(s)=t} \pi_s = 1$ .

## Households

single representative household consumes  $c_s \in \mathfrak{R}_+$

lifetime utility of  $\sum_{s \in S} \delta^{t(s)} u(c_s) \pi_s$ ;  $0 \leq \delta < 1$

period utility function is smooth, concave, bounded below

for levels of consumption above subsistence  $c \geq \underline{c}$  is CES  $u(c) = -(1/\theta)c^{-\theta}$

[ $\theta \geq 0$  the CES function not bounded below: why we make subsistence assumption]

## *Production Possibilities*

countable *types* of capital  $\omega = 0, 1, \dots$

each type potentially available in countable *generations*  $i = \dots, -1, 0, 1, \dots$

$k^{\omega i}$  amount of type  $\omega$  capital of generation  $i$

type/generation pair called “kind” of capital

linear activities, using capital as input

input one unit of capital of particular kind gives a single output

- 1)  $\gamma^i$  units of the consumption characteristic,  $\gamma > 1$
- 2)  $\beta > 1$  units of the same kind of capital
- 3)  $\rho: \beta > \rho > 1$  next generation of capital
- 4) 1 unit next type of capital with loss of  $L$  generations

$$\gamma\rho > \beta$$

better to move to next generation of capital than stay with current generation

never want to move to next type

only finite subset of activities available at a moment of time

state:

- latest type of capital
- latest generation for each available type
- number of periods since latest type of capital introduced

for less than the latest type of capital can't produce more than latest generation

cannot produce past the latest type of capital

“good times” can produce next generation of highest type: state  $\eta_g$

“bad times” can no longer produce more generations of current type –  
new type becomes available: state  $\eta_b$

many parallel capital ladders representing different technologies  
(jet planes, computers...)

want to move up the ladder by incremental improvements

however, sometimes technologies become exhausted, no further  
incremental improvements are possible, and must move to the next  
ladder, losing  $L$  generation in the process

$1 - \pi$  probability of continuing in  $\eta_g$

$\pi$  probability of reverting to  $\eta_b$

except that for  $M$  periods after  $\eta_b$  occurs, only  $\eta_g$  can occur

- leads to computational simplicity  
makes it hard to match real data

## *Equilibrium and Prices*

look at solutions to problem of maximizing consumer utility subject to technology constraints

prices are efficiency prices

## Analysis of the Model

### Basic Results

single unit generation 0 capital is stalled

produce output  $t > 1$  periods from now

- build more of existing type and generation of capital until period  $t - 1$   
yields  $\beta^{t-1}$  units of characteristic
- build unit of the next type of capital of generation  $-L$   
use to build generation  $-L + 1$  etc until period  $t - 1$   
yields  $\rho^{t-2} \gamma^{t-2-L}$  units of characteristic

$$m^* = 1 + \frac{(L+1)\log \gamma + \log \rho}{\log \rho + \log \gamma - \log \beta}$$

for  $t < m^*$  use old kind of capital

for  $t > m^*$  use new kind of capital

**Proposition 4.1:**  $M > m^* - 1$  initial stock of capital is most advanced kind then with probability 1 in the future

(1) a positive amount of the most advanced kind of capital

(2) at most one other kind of capital, of the type immediately preceding most advanced kind

(3) bad state occurs only when there is a single type of capital

$z(\eta)$  most advanced kind of capital in state  $\eta$

$V(\eta_s, k_s^{z(\eta_s)})$  p.v. of future utility beginning at  $\eta_s$  with with  $k_s^{z(\eta_s)}$  units of the most advanced capital only

$m(\eta)$  is number of periods since last bad state

$V_m$  value of a single unit of generation 0 capital in state where  $m(\eta) = m$  for  $m > M$  that  $V_m = V_M$

let  $\bar{i}$  be generation of the most advanced kind of capital at  $\eta_s$

$$V(\eta_s, k_s^{z(\eta_s)}) = \left( \gamma^{\bar{i}} k_s^{z(\eta_s)} \right)^{-\theta} V_{m(\eta_s)}$$

capital produced for period  $t$  before the state at period  $t$  is known  
used in period  $t$  after the state is determined

compare value of the capital stock (relative to the price of consumption)  
at  $t$  immediately before and immediately after the state is realized

good news has a marginal impact on the value of capital, but bad news  
causes it to change abruptly

**Proposition 4.2:**  $M > m^* - 1$ , initial stock of capital only most advanced kind

(1)  $\theta < 0$ , news of a negative shock  $m(\eta) = 0$  causes the value of the capital stock to fall immediately

(2)  $\theta > 0$  the value of the capital stock rises immediately

(3) ratio of the value of capital stock after bad news to that before

$$\pi + (1 - \pi) \frac{V_0}{V_M}$$

(4) ratio of the value of capital stock after bad news to that before

$$(1 - \pi) + \pi \frac{V_M}{V_0}$$

time intervals short, so  $\pi$  close to zero (bad state unlikely) then  
ratio on bad news =  $V_0 / V_M$ ; after good news = 1

once the good state is sufficiently well established [ $m(\eta) \geq M$ ]  
value of the capital stock from period to period rises exponentially

can explicitly solve for interim period

## Quantitative Aspects of the Theory

$\pi \rightarrow 0$

new technologies difficult to get on line:  $L \rightarrow \infty$   
gives most extreme stock market fall

$g$  growth rate of consumption in good state

$$\frac{V_0}{V_M} = \frac{\left(1 - \left(\delta^{\frac{1-\theta}{1+\theta}} g^{-\theta}\right)\right)^{\theta+1}}{\left(1 - \left(\delta^{\frac{1-\theta}{1+\theta}} g_b^{-\theta}\right)\right)^{\theta+1}}$$

as logarithmic case approached [ $\theta \rightarrow 0$ ]  $V_0 / V_M \rightarrow 1$

as  $\theta \rightarrow -1$  (utility approaches linearity)  $V_0 / V_M \rightarrow 1$

$g_b = 1$  no growth following negative shock  
 market rate of interest in bad state 5%  
 growth of consumption in good state 5%

$\theta$	-0.1	-0.2	-0.4	-0.8
$1 - V_0 / V_M$	6.8%	9.9%	9.6%	1.4%
Interest at $M$	9.1%	8.6%	7.7%	5.9%
Interest at 0	5.0%	5.0%	5.0%	5.0%

intermediate range of  $\theta$  market drops roughly 10%

$g_b = 1$  no growth following negative shock  
 market rate of interest in bad state 5%  
 growth of consumption in good state 10%

$\theta$	-.1	-.2	-.4	-.8
$1 - V_0 / V_M$	13.4%	19.8%	19.7%	2.9%
Interest at $M$	12.8%	12.0%	10.3%	6.8%
Interest at 0	5.0%	5.0%	5.0%	5.0%

market drops on the order of 20%

5% interest in bad state high by historical standards

$g_b = 1$  no growth following negative shock

market rate of interest in bad state 5%

growth of consumption in good state 5%

$\theta$	-.1	-.2	-.4	-.8
$1 - V_0 / V_M$	17.8%	26.4%	27.8%	2.9%
Interest at $M$	6.2%	5.8%	4.8%	4.4%
Interest at 0	2.0%	2.0%	2.0%	3.0%

nearly 30% drop in the market

$g_b = 1$  no growth following negative shock

market rate of interest in bad state 5%

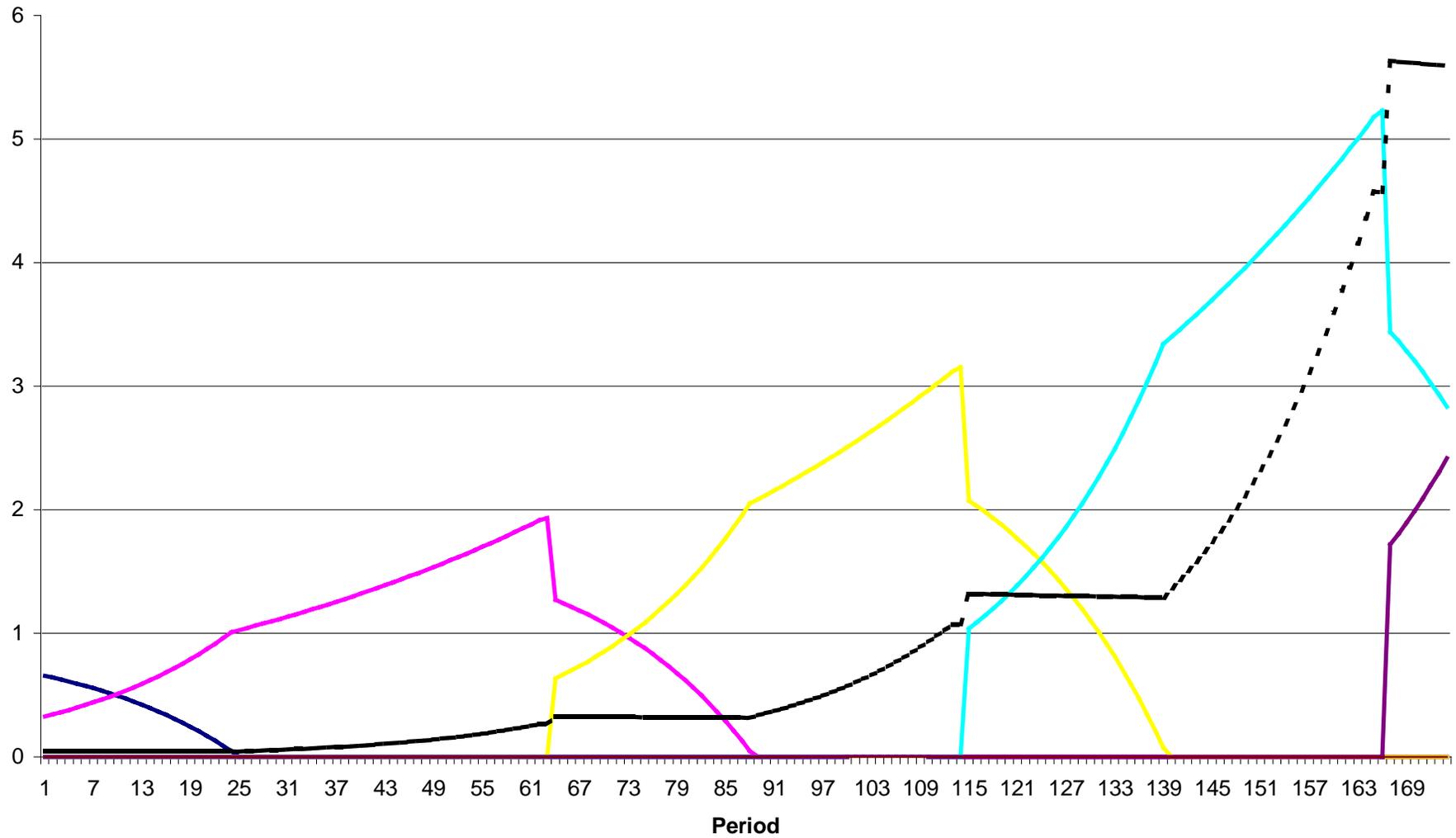
growth of consumption in good state 5%

$\theta = -.4$

$\pi = 0.05, M = 12, L = 20, \beta = 1.052, \rho = 1.05, \gamma = 1.032$

$\theta$	approximation	exact
$1 - V_0 / V_M$	9.6%	9.0%
$g$	5.0%	5.0%
$g_b$	0.0%	0.0%

**Capital Stock**  
dotted line is consumption



## ***Conclusions***

### *Limitations*

- No explicit labor supply rule
- Only large and infrequent shocks provoked my major technological change
- Only two types of capital are in use at each moment of time
- New generation becomes available just when old one stalls out