Collateral and Bankruptcy in Debt Constrained Markets

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August 29, 2006

An Econ Ceremony

a rational complete market model of bankruptcy and collateral

contrast the incomplete markets bankruptcy models of Dubey, Geanakoplos, and Shubik [1988] and Zame [1993].

The Modl

"little or no practical use"

an infinite number of discrete time periods $t = 0,1,\ldots$

each period two types of consumers $i = 1, 2$

a continuum of each type of consumer

each moment of time, one consumer has high productivity and one low productivity

state $\eta_t \in \{1,2\}$ at time t indexes consumer who has high productivity

random variable follows Markov process characterized by probability of a *reversal* where two types switch productivity $0 < \pi < 1$

Totem of the Math-Econ

venturing stark naked into the chill winds of abstraction

economy takes place on a tree

root of tree determined by the fixed initial state η_0

state history a finite list $s = (\eta_1, \ldots, \eta_t)$ of events that have taken place through time $t(s)$

history immediately prior to s denoted $s - 1$

if node σ follows s on the tree, we write $\sigma > s$

countable set of all state histories is denoted S

probability of state history computed from the Markov transition probabilities

$$\pi_s = pr(\eta_{t(s)} \mid \eta_{t(s)-1}) pr(\eta_{t(s)-1} \mid \eta_{t(s)-2}) \cdots pr(\eta_1 \mid \eta_0).$$

single consumption good x

representative consumer of type i consumes x_s^i in state history s

common stationary additively separable expected utility

$$(1-\delta)\sum_{s\in S}\delta^{t(s)}\pi_s u(x_s^i).$$

period utility function twice continuously differentiable with $Du(x) > 0$

satisfies the boundary condition $Du(x) \to \infty$ as $x \to 0$

$D^2u(x) < 0$

common discount factor δ satisfies $0 < \delta < 1$

types of capital

human capital (or labor)

services of the single unit of human capital held by type i consumer in state η is denoted $w^i(\eta)$

takes on one of two values, ω^b and ω^g, with $\omega^b < \omega^g$, corresponding to low and high productivity respectively

if one consumer has high productivity other consumer low so $w_t^i = \omega^b$ means $w_t^{-i} = \omega^g$ ($-i$ is the type of consumer who is not type i).

state indexes which consumer has high productivity, so $\omega^\eta(\eta) = \omega^g, \omega^{-\eta}(\eta) = \omega^b$.

physical capital (trees or land).

one unit of physical capita

durable and returns $r > 0$ of the consumption good every period

interpret physical capital as trees, with r being the amount of consumption fruit produced every period by the trees

consumer of type i holds a share θ_s^i of the capital stock contingent on the state history s

initial physical capital holdings are θ_0^i.

total supply of the consumption good in this economy

sum of the individuals' productivity, plus return on single unit of physical capital

$\omega \equiv \omega^g + \omega^b + r$

social feasibility conditions for this economy in each history

$$x_s^1 + x_s^2 \leq \omega^g + \omega^b + r = \omega$$

$$\theta_s^1 + \theta_s^2 \leq 1.$$

Debt Constrained Economy

borrowing, lending and the sale and purchase of insurance contracts are possible

however: debt constraints because consumers can opt out of intertemporal trade

renege on all existing debts and excluded forever from all further participation in intertemporal trade

however physical capital is seized

endowment of human capital inalienable: cannot be taken away, nor can consumers be prevented from consuming its returns.

according to the totem consumers face an individual rationality constraint

$$(1-\delta)\sum_{\sigma \geq s} \delta^{t(\sigma)-t(s)} (\pi_\sigma / \pi_s) u(x_\sigma^i) \geq (1-\delta)\sum_{\sigma \geq s} \delta^{t(\sigma)-t(s)} (\pi_\sigma / \pi_s) u(w^i(\eta_\sigma)).$$

in every state history value of continuing to participate economy no less than value of dropping out markets are complete, consumers purchase contingent consumption for the state history s for the present value price p_s and they sell the return on their capital $w^i(\eta_s) + r\theta_0^i$ at the same price

corresponding optimization problem is

$$\max\ (1-\delta)\sum\nolimits_{t=0}^{\infty} \delta^t u(x_t^i)$$

subject to

$$\sum\nolimits_{s\in S} p_s x_s^i \le \sum\nolimits_{s\in S} p_s(w^i(\eta_s) + \theta_0^i r)$$

$$(1-\delta)\sum\nolimits_{\sigma\ge s} \delta^{t(\sigma)-t(s)}(\pi_\sigma / \pi_s)u(x_\sigma^i) \ge (1-\delta)\sum\nolimits_{\sigma\ge s} \delta^{t(\sigma)-t(s)}(\pi_\sigma / \pi_s)u(w^i(\eta_\sigma)).$$

Equivalent Sequence of Securities Markets

$$x_s^i + q_{(s,1)}\theta_{(s,1)}^i + q_{(s,2)}\theta_{(s,2)}^i \leq w^i(\eta_s) + (v_s + r)\theta_s^i$$

$$\theta_s^i \geq -\Theta, \quad \theta_0^i \text{ fixed},$$

$q_{(s,\eta)}$ is price of the Arrow security traded in state s that promises a unit of physical capital to be delivered at state (s,η)

standard arbitrage argument implies that $q_{(s,1)} + q_{(s,2)} = v_s$

constraint $\theta_s^i \geq -\Theta$ rules out Ponzi schemes where Θ is a positive constant chosen large enough not to constrain to borrowing.

equilibrium of *the debt constrained economy*

infinite sequence of consumption levels and consumption prices

- consumers maximize utility given their constraints
- social feasibility condition for consumption is satisfied

symmetric stochastic steady state

consumption x^g when productivity is high, x^b when productivity is low, and the rule

$$x_s^i = \begin{cases} x^g & w_s^i = \omega^g \\ x^b & w_s^i = \omega^b. \end{cases}$$

Solution of the Debt Constrained Model

find symmetric steady state by decreasing x^g from ω^g until we either achieve the symmetric first best at $x^g = \omega/2$ or until individual rationality constraint begins to bind

define a function proportional to the difference between the utility from the steady state consumption plan and consumption in autarky

recursive calculation shows that this function is

$$f^D(x^g) = \left[1 - \delta(1-\pi)\right]\left[u(x^g) - u(\omega^g)\right] + \delta\pi\left[u(\omega - x^g) - u(\omega^b)\right].$$

Proposition 1: A symmetric stochastic steady state x^g of the debt constrained economy is characterized by

$f^D(\omega/2) \geq 0$ and $x^g = \omega/2$ or

$\omega^g > \omega/2$, $f^D(x^g) = 0$ and $x^g \in [\omega/2, \omega^g]$

when $0 < \pi < 1$ then f^D is concave and satisfies $f^D(\omega^g) > 0$, leading to the conclusion

Proposition 2: A symmetric stochastic steady state exists in the debt constrained economy. There is only one symmetric stochastic steady state.

The Collateral Economy

call the present value of future consumption at equilibrium prices virtual wealth

each type consumes an amount either x^g or x^b today

virtual wealth less consumption grows by $1/\Delta$ where Δ is the market discount factor

virtual wealth then has to be adjusted based on the state next period to reflect the present value of consumption starting in that state

Loans and Collateral

no contingent claims, only loans

loan may also be collateralized – with an amount of collateral in the form of physical capital C

bankruptcy is allowed

- in bankruptcy you pay the collateral and not the loan
- bankruptcy is allowed only if you can "prove" that paying off the loan would cause you to "run"
- you must provide collateral if you are in a "risk" class to which no one would lend if you didn't

Implementation of the Second Best

today each type makes the other type a loan: loans are of equal size

whichever type has to increase virtual wealth next period should default on their loan: they collect the promised payment from the other type, but they do not pay back their own loan.

however, the size of the payment depends on whether or not a reversal takes place

there must be two different transfers: when there is a reversal and when there is no reversal

off-setting loans cover the larger of the two amounts

type that will receive the lower payment next period must collateralize an amount equal to the difference in the two payments

when they go bankrupt they pay the collateral rather than zero, offsetting the amount that they receive in repayment of their own loan.

high endowment type can not be prevented from lending giving the market discount factor

$$\Delta = \delta \frac{(1-\pi)u'(x^g) + \pi u'(x^b)}{u'(x^g)}.$$

recursive relations for virtual wealth

$$W^g = x^g + \Delta[(1-\pi)W^g + \pi W^b]$$

$$W^b = x^b + \Delta[(1-\pi)W^b + \pi W^g]$$

solved to find

$$W^g = \frac{1}{1-\Delta} \frac{x^g \left[1-\Delta(1-\pi)\right] + \Delta\pi x^b}{\left[1-\Delta(1-\pi)\right] + \Delta\pi}$$

$$W^b = \frac{1}{1-\Delta} \frac{x^b \left[1-\Delta(1-\pi)\right] + \Delta\pi x^g}{\left[1-\Delta(1-\pi)\right] + \Delta\pi}$$

amount of payment that a type pays to have the correct virtual wealth next period

$$(1/\Delta)(W^g - x^g) - W^g = (1/\Delta)[(1-\Delta)W^g - x^g] = -\frac{\pi(x^g - x^b)}{(1-\Delta(1-\pi)) + \Delta\pi}$$

$$\begin{aligned}(1/\Delta)(W^b - x^b) - W^g &= (1/\Delta)((1-\Delta)W^b - x^b + \Delta(W^b - W^g)) \\ &= -\frac{(1-\pi)(x^g - x^b)}{(1-\Delta(1-\pi)) + \Delta\pi}\end{aligned}$$

assume $\pi \leq 1/2$

difference between two payments (with and without a reversal)

$$C = \frac{(1-2\pi)(x^g - x^b)}{(1-\Delta(1-\pi)) + \Delta\pi}$$

which has to be the amount of collateral

both loan the other the larger amount

$$\frac{(1-\pi)(x^g - x^b)}{(1-\Delta(1-\pi)) + \Delta\pi}$$

and the person currently in the good state collateralizes C of his borrowing. Whoever winds up in the good state goes bankrupt.

present value of collateral in the economy is

$$\frac{r}{1-\Delta} \geq C$$

Leijonhufvudian Economics and the Economics of Leijonhufvud

The deep Leijonhufvudian insight: modern economies work pretty well on a day-to-day basis

A Leijonhufvudian should not have much sympathy for the incomplete market model – which attempts to explain why the economy works poorly on a day-to-day basis

This model explains why institutions such as bankruptcy help the economy work well on a day-to-day basis

The model can form the basis for the true Leijonhufvudian question: how might these institutions break if the shock is too large? This is the question that now needs to be addressed by a macroeconomic theory that can now give a pretty good account of the “ordinary” working of the economy

- underdevelopment: too little collateral?
- Credit Chains and Fragility

  the mechanism designed to support day-to-day transactions

  such a decentralization may “break” when faced with unusual shocks – leading to unanticipated bankruptc

observations about the econ tribe

1. "They are poor – except for a tiny minority miserably poor."

2. "The adult econ used to regard himself as a lifelong member of his dept. This is no longer true."

"Under circumstances such as these, we expect alienation, disorientation and a general loss of spiritual values."

Actually, the #2 seems to have solved #1, so perhaps the supply and demand totem works after all