Long Run versus Short Run Player

Moral Hazard

choose $a^i \in A$

observe $y \in Y$

$\rho(y|a)$ probability of outcome given action profile

$u^i(a^i, y)$ utility

private history: $h^i = (a_1^i, a_2^i, \ldots)$

public history: $h = (y_1, y_2, \ldots)$

strategy $\sigma^i(h^i, h) \in \Delta(A^i)$

“public strategies” , *perfect public equilibrium*

Moral Hazard Example

mechanism design problem

each player is endowed with one unit of income

players independently draw marginal utilities of income $\eta \in \{\overline{\eta}, \underline{\eta}\}$

player 2 (SR) has observed marginal utility of income

player 1 (LR) has unobserved marginal utility of income

player 2 decides whether or not to participate in an insurance scheme player 1 must either announce his true marginal utility or he may announce $\overline{\eta}$ independent of his true marginal utility

non-participation: both players get $\gamma = \dfrac{\overline{\eta} + \underline{\eta}}{2}$

participation: the player with the higher marginal utility of income gets both units of income normal form

	non-participation	participate
truth	γ, γ	$\frac{\bar{\eta}+\gamma}{2}, \frac{\bar{\eta}+\gamma}{2}$
lie	γ, γ	$\frac{3\gamma}{2}, \frac{\bar{\eta}}{2}$

$p^* = \frac{\eta}{\gamma}$ makes player 2 indifferent max $u^1(a) = \frac{3\gamma}{2}$

mixed precommitment/Stackelberg= $\frac{\overline{\eta}+\gamma}{2} + (1 - \frac{\underline{\eta}}{\gamma})\frac{\underline{\eta}}{2}$

$\overline{v}^1$ best dynamic equilibrium= $\frac{\overline{\eta}+\gamma}{2}$

pure precommitment/Stackelberg= $\frac{\overline{\eta}+\gamma}{2}$

Set of dynamic equilibria

static Nash= γ

$\underline{v}^1$ worst dynamic equilibrium= γ

min $u^1(a) = \gamma$, minmax= γ

moral hazard case

player 1 plays “truth” with probability p^* or greater

player 2 plays “participate”

$$\bar{v} = (1-\delta)\frac{\bar{\eta}+\gamma}{2} + \delta\left(\frac{1}{2}w(\underline{\eta}) + \frac{1}{2}w(\bar{\eta})\right)$$

$$\bar{v} \geq (1-\delta)\frac{3\gamma}{2} + \delta w(\bar{\eta})$$

$$\bar{v} \geq w(\underline{\eta}), w(\bar{\eta})$$

$w(\bar{\eta})$ must be as large as possible, so inequality must bind; $w(\underline{\eta}) = \bar{v}$

$$\bar{v} = (1-\delta)\frac{3\gamma}{2} + \delta w(\bar{\eta})$$

solve two equations

$$\bar{v} = \bar{\eta} - \frac{\gamma}{2}$$

$$w(\bar{\eta}) = \frac{\bar{v} - (1-\delta)3\gamma/2}{\delta}$$

check that $w(\overline{\eta}) \geq \gamma$

leads to $\delta \geq 2\left(2 - \frac{\overline{\eta}}{\gamma}\right)$

from $\delta < 1$ this implies

$\overline{\eta} > 3\underline{\eta}$

Moral Hazard Mixing Games

$\rho(y \mid a) > 0$

then $\overline{v}_{MH} < \overline{v}_{PO}$

moral hazard worse than perfect observability