Econ 504 (2008) Microeconomic Theory II - Midterm I
2/13/2008

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Instructions: This is an open book exam, you can use any written material. You have 1 hour 20 minutes. The weight of each question is indicated next to it. Good luck!

1. (30pts) Consider the signalling game in the last page. Find all of its the sequential equilibria: pooling, separating, and those involving mixed strategies. Indicate which pooling equilibria satisfy the intuitive criterion.

2. (25pts) Consider the following two player normal form game. Two firms 1 and 2 simultaneously chose prices p_1 and p_2 in $[0, 1]$. The profit of firm i is given by $\pi_i(p_i, p_j) = p_i(1 + p_j - p_i)$ where $j \neq i$.

   (a) Assume that: (i) firm 1 is rational, (ii) firm 1 knows that firm 2 is rational, (iii) and firm 1 knows that firm 2 knows that firm 1 is rational. What can you conclude about the range of prices that firm 1 might set?

   (b) Assume that rationality of both firms is common knowledge. What can you conclude about the prices set by each firm? What does your finding imply about Nash equilibria.

3. Consider an infinite horizon bargaining game with exit among three players $N = \{1, 2, 3\}$. Players maximize discounted expected payoffs and have the common discount factor $\delta \in (0, 1)$.

   In the beginning of a period t, if no player has exited before, then one of the three players is randomly selected with equal probability 1/3 to make an offer. The selected player i offers a division of the cake (x_1, x_2, x_3) where $x_1, x_2, x_3 \geq 0$ and $x_1 + x_2 + x_3 = 1$ (x_l denotes player l's share for $l = 1, 2, 3$). The two other players j and k observe i's offer (x_1, x_2, x_3), then j and k simultaneously accept or reject this offer. If both j and k reject, then all three players move to period $t + 1$. If both j and k accept then the division is carried out and the game is over with payoffs $(\delta^{t-1}x_1, \delta^{t-1}x_2, \delta^{t-1}x_3)$. If k accepts but j rejects, then k receives a payoff of $\delta^{t-1}x_k$ and exits the game, players i and j move to period $t + 1$ and continue the bargaining process with the remainder cake $\alpha = 1 - x_k$.

   If in the beginning of a period t, there are only two remaining players i and j and a cake of size $\alpha \geq 0$, then one of them is randomly selected with equal probability

1/2 to make an offer. The selected Player offers a division of the cake (x_i, x_j) where $x_i, x_j \geq 0$ and $x_i + x_j = \alpha$. The other player observes the offer, then decides to accept or reject it. If the offer is accepted then the division is carried out and the game is over with payoffs $(\delta^{t-1}x_i, \delta^{t-1}x_j)$ to i and j. If the offer is rejected then, i and j move to period $t+1$ and continue bargaining over the cake of size α.

Players who never exit the game and never reach an agreement receive zero payoff. The process of who makes an offer is independent across periods.

(a) (15pts) Consider a subgame starting with only two remaining players i and j and a cake of size α. Derive SPE strategies for such a subgame. Show that the strategies you derived indeed form an SPE.

(b) (30pts) Derive SPE strategies for the original game with all three players, using what you have found in part (a). Show that the strategies you derived indeed form an SPE.

A Signaling Game