Economic 211, David K. Levine
Answers to Problems on Learning

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1. in a 2x2 game a marginal best best response distribution is a correlated equilibrium: weak marginal best response must satisfy that actual utility is greater than the utility from playing either action against the marginal; this means

$$u_{11}\rho_{11} + u_{12}\rho_{12} + u_{21}\rho_{21} + u_{22}\rho_{22} \geq u_{11}(\rho_{11} + \rho_{21}) + u_{12}(\rho_{12} + \rho_{22})$$
$$u_{11}\rho_{11} + u_{12}\rho_{12} + u_{21}\rho_{21} + u_{22}\rho_{22} \geq u_{21}(\rho_{11} + \rho_{21}) + u_{22}(\rho_{12} + \rho_{22})$$

rearranging each inequality gives

$$u_{21}\rho_{21} + u_{22}\rho_{22} \geq u_{11}\rho_{21} + u_{12}\rho_{22}$$
$$u_{11}\rho_{11} + u_{12}\rho_{12} \geq u_{21}\rho_{11} + u_{22}\rho_{12}$$

which is in fact the condition for a correlated equilibrium: each action should be a best response against the conditional for that action

2. with an initial condition where both players observe one heads, find first ten periods of fictitious play in matching pennies and frequency each player played heads

	1	2	3	4	5	6	7	8	9	10
match	H	H	T	T	T	T	T	T	T	T
opp	T	T	T	T	T	T	H	H	H	H

P1 plays heads 2/10; P2 plays heads 4/10; p1 wins 4/10

3. Consider a two state Markov process in which there is a 3/4 chance of remaining in state 1 but only a 1/2 chance of remaining in state 2. What is the unique stationary distribution? What does this mean about the long run frequency with which state 1 is observed?

transition matrix $P = \begin{pmatrix} 3/4 & 1/2 \\ 1/4 & 1/2 \end{pmatrix}$; stationary transition probabilities $Px = x$ means $\frac{3}{4}x_1 + \frac{1}{2}x_2 = x_1$ so $x_1 = 2/3, x_2 = 1/3$, so in the long run state 1 is observed 2/3 of the time.

4. If the second node is reached with positive probability player 2 plays DOWN. However, player 1 does better playing across in this case, so the second node is NOT

reached with positive probability. The only heterogeneous-self-confirming equilibrium is ACROSS, with player 2 playing anything at all.

5. (a) (1/3,1/3,1/3)

(b) We use the substitution $\theta_3 = 1-\theta_1-\theta_2$ to reduce the system to a two dimensional system. Note also that the average utility is given by

$$\theta_1^2+\theta_2^2+(1-\theta_1-\theta_2)^2+2\theta_1\theta_2+2\theta_2(1-\theta_1-\theta_2)+2\theta_1(1-\theta_1-\theta_2)=$$

$(\theta_1+\theta_2+(1-\theta_1-\theta_2))^2=1$ independent of θ_1,θ_2.

So the replicator dynamic is

$$\dot{\theta}_1=\theta_1(\theta_1+2\theta_2-1)$$
$$\dot{\theta}_2=\theta_2(\theta_2+2(1-\theta_1-\theta_2)-1)$$

Differentiating, the derivative matrix is $\begin{bmatrix} 1/3 & 2/3 \\ -2/3 & -1/3 \end{bmatrix}$ and the eigenvalues are the roots of $(1/3-\lambda)(-1/3-\lambda)+4/9=0$ or $\pm\sqrt{\frac{1}{3}}i$

(c) This doesn't tell much, except that the system may cycle. The steady state is neither asymptotically stable nor unstable.

6. $\dot{\theta}_i(s_i)=\dfrac{\exp(\kappa_i u_i(s_i,\theta_{-i}))}{\sum_{\tilde{s}_i}\exp(\kappa_i u_i(\tilde{s}_i,\theta_{-i}))}-\theta_i(s)$

(a) approaches b.r. dynamic

(b) $d\dot{\theta}_i(s_i)/d\theta_i(s_i)=-1$ so the matrix $D_{\theta_i}\dot{\theta}_i$ has diagonal equal to -I or trace equal to -*m* the number of strategies. This implies (by Liouville's theorem) that the dynamical system is volume contracting.

(c) In general volume contraction means that the system in the long-run remains on a manifold (surface) with dimension one less than the total dimension of the system. In particular with two players and two actions, the overall system is two-dimensional so must in the long run move on a curve. This in turn implies (in continuous time) convergence to a steady state (no cycles).