Carlsson and van Damme

	Invest	NotInvest
Invest	θ,θ	$\theta-1,0$
NotInvest	$0,\theta-1$	0,0

Three cases

- $\theta > 1$ dominant strategy to invest
- $\theta \in [0,1]$ two pure equilibria – coordination problem
- $\theta < 1$ dominant strategy not to invest

incomplete information about θ

each player observes a noisy signal $x_i = \theta + \sigma\varepsilon_i$

where ε_i are independent normal random variables with zero mean and unit variance

improper uniform prior over θ

each player sees θ as normal with mean x_i and variance σ^2; each sees their opponents signal as the sum of this normal and an independent normal with mean zero and variance σ^2, that is, a normal with mean x_i and variance $2\sigma^2$

expected utility gain from investing if probability of opponent notInvesting is $q(x_i)$ is

$E[\theta|x_i] - q(x_i)$ so best response is invest if this is non-negative; since $E[\theta|x_i] = x_i$ this can be written as $x_i - q(x_i)$

Suppose you believe your opponent invests for $x_{-i} > b$. Then

$q(x_i) \leq \Phi(-(b - x_i)/(2^{1/2}\sigma))$, hence you must invest if

$x_i > \Phi(-(b - x_i)/(2^{1/2}\sigma))$

Suppose you believe your opponent notInvests for $x_{-i} < b$. Then

$q(x_i) \geq \Phi(-(b - x_i)/(2^{1/2}\sigma))$ hence you notInvest if

$x_i < \Phi(-(b - x_i)/(2^{1/2}\sigma))$

Implicitly define the function $b(k) = \Phi(-(b(k) - k)/(2^{1/2}\sigma))$

this has a unique solution because lhs strictly increasing in b and rhs strictly decreasing in b

since rhs strictly increasing in k, $b(k)$ is strictly increasing

$b(k)$ has a unique fixed point at ½

why?? substitute $b(k) = k$ and the RHS becomes $\Phi(0) = 1/2$.

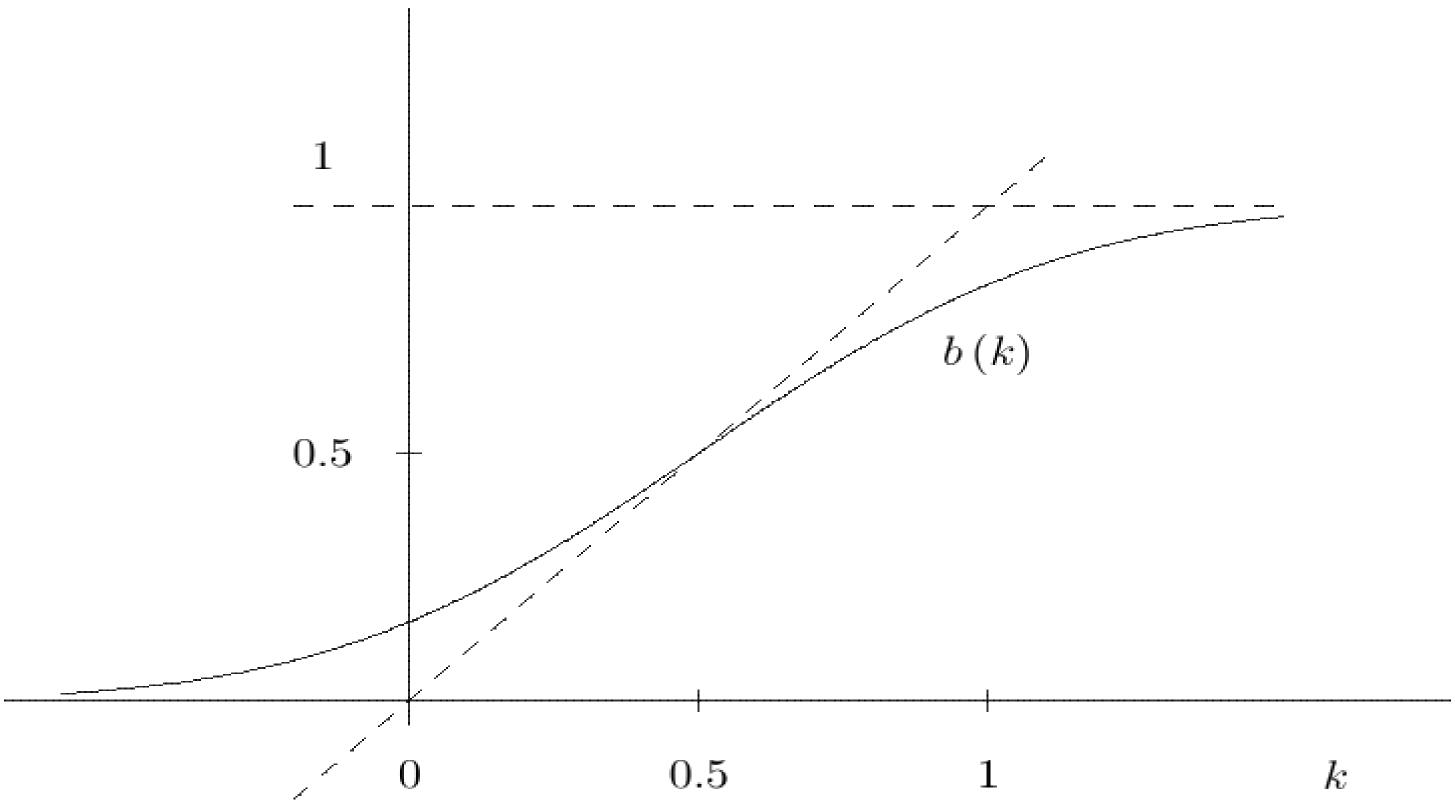

Figure 2.1: Function $b(k)$

any strategy that is not dominated must satisfy

$$s(x) = \begin{cases} \text{Invest} & x > 1 \\ \text{NotInvest} & x < 0 \end{cases}$$

suppose you know your opponent will choose NotInvest for $x < k$
dominance implies you should choose NotInvest for $x < b(k)$

suppose you know your opponent will choose Invest for $x > k$
dominance implies you should choose Invest for $x > b(k)$

so after n round of iterated dominance

$$s(x) = \begin{cases} \text{Invest} & x > b^n(1) \\ \text{NotInvest} & x < b^n(0) \end{cases}$$

Since $b(k)$ strictly increasing and has a unique fixed point at $1/2$

$\lim_{n\to\infty} b^n(0), b^n(1) = 1/2$ (see the diagram)

so the only thing to survive iterated weak dominance is the cutpoint strategy

$$s(x) = \begin{cases} \text{Invest} & x > 1/2 \\ \text{NotInvest} & x \leq 1/2 \end{cases}$$

and this is a best response to itself since $b(1/2) = 1/2$ so it is and equilibrium as well as the only thing to survive iterated dominance

(weak or strong dominance?)

conditional on θ the choice of the two players is independent and the probability of investment is

$$\Phi((\frac{1}{2}-\theta)/\sigma)$$

also a continuum of players result:

payoff to investing $\theta - 1 + l$ where l is fraction of players investing

iterated deletion of dominated strategies leaves only: Invest when you get a signal greater than $1/2$.

relationship to common knowledge