# **The Folk Theorem**

**Review of Long Run vs. Short run** 

+  $\max u^{1}(a)$ mixed precommitment/Stackelberg  $\overline{v}^{1}$  best dynamic equilibrium best dynamic equilibrium w/ moral hazard pure precommitment/Stackelberg Set of dynamic equilibria static Nash worst dynamic equilibrium w/ moral hazard  $\underline{v}^{1}$  worst dynamic equilibrium minmax - min  $u^{1}(a)$ 

- structure of an equilibrium
- role of reputation (can do strictly better when there is moral hazard)

## Simple Folk Theorems

- socially feasible
- individually rational

Statement of Folk Theorem

Prisoner's Dilemma Game

	R	L
U	2,2	0,3
D	3,0	1,1

- Nash with time averaging
- perfect Nash threats with discounting

public randomization vs. discount factors near one

$$v_t = (1 - \delta)u_t + \delta v_{t+1}$$
$$v_{t+1} = \delta^{-1}v_t - (1 - \delta)\delta^{-1}u_t$$

note that coefficient add up to one

#### Fudenberg Maskin Theorem

issue: perfection and minmaxing

minmax followed by reversion to another equilibrium note simultaneous determination of equilibria

## Matching and Information Systems (Kandori)

 $u^i(a)$ 

*I* a finite set of information states  $\eta: A \times I^2 \to I$  an information system if at *t* you and your opponent played  $a_t$  and had states  $\eta_t^i, \eta_t^{-i}$ , then your next state is  $\eta_{t+1}^i = \eta(a_t, \eta_t^i, \eta_t^{-i})$ 

players randomly matched in a population observe their current opponents current state

Ellison: even without information states could have cooperation due to contagion effects

But contagion effects diminish as population size grows

Folk Theorem for information systems: socially feasible individually rational payoff – exists an information system that supports it

# Example

Prisoner's dilemma

	С	D
С	x, x	0, x + 1
D	<i>x</i> +1,0	1,1

$$I = \{r, g\} \\ \eta(a^{i}, \eta^{-i}) = \begin{cases} G & (a^{i}, \eta^{-i}) = C, G \\ R & (a^{i}, \eta^{-i}) = C, R \\ R & (a^{i}, \eta^{-i}) = D, G \\ G & (a^{i}, \eta^{-i}) = D, R \end{cases}$$

"green team strategy" defect on red cooperate on green

$$V(g) = x$$
  

$$V(r) = \delta x$$
  

$$C (1-\delta)x + \delta V(g) = x$$
  

$$D \frac{(1-\delta)(x+1) + \delta V(r) = (1-\delta)(x+1) + \delta^{2}x = (1-\delta) + (1-\delta+\delta^{2})x$$
  

$$x \ge (1-\delta) + (1-\delta+\delta^{2})x$$
  

$$So \ \delta(1-\delta)x \ge (1-\delta)$$
  

$$\delta \ge 1/x$$

Remark: this method works also with finitely-lived individuals, although naturally towards the end of their life, you may not be able to punish them very much © This document is copyrighted by the author. You may freely reproduce and distribute it electronically or in print, provided it is distributed in its entirety, including this copyright notice. Source: <u>\DOCS\Annual\99\class\grad\FOLK SLIDES.DOC</u>