1. Short Answers

For each of the normal form games below, find all of the Nash equilibria. Which are Pareto Efficient?

a)

	L	R
U	1,0	3*,1*(Efficient)
D	2*,2*(Efficient)	1,0

Nash Equilibria:

(D,L): Efficient
(U,R): Efficient

b)

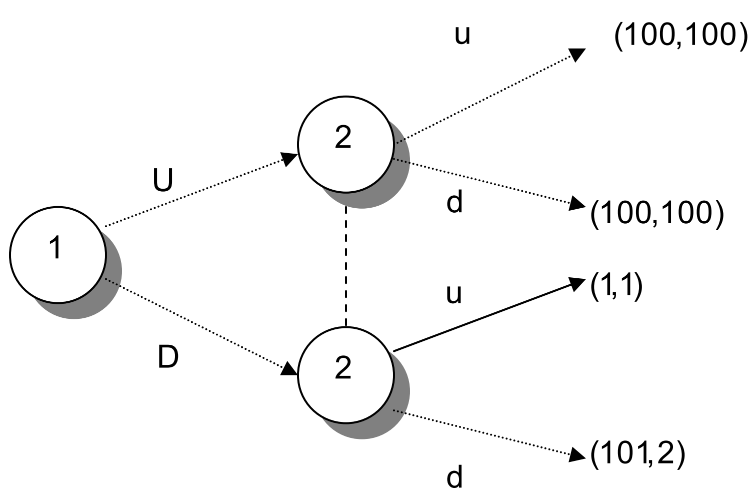
	L	R
U	2*,5* (Not efficient)	6*,3
D	0,9*	4,7

Nash Equilibrium:

(U,L): Not efficient

For each of the extensive form games below, find the normal form and all Nash equilibria. Then find all of the subgame perfect equilibria. Which are Pareto Efficient?

c) Extensive form with subgame perfect choices marked with dashed lines



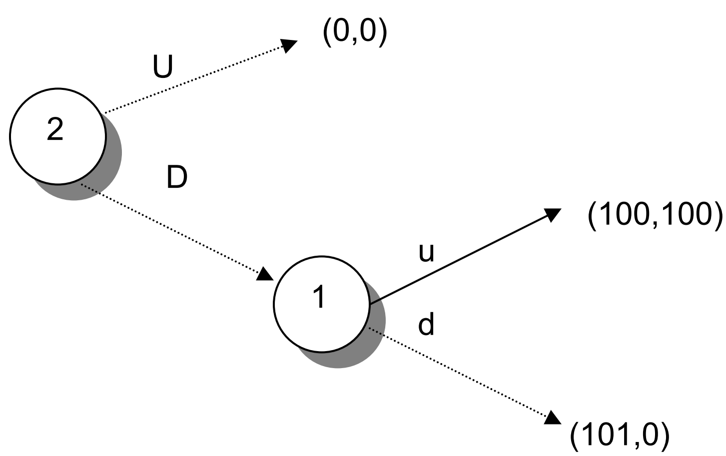
normal form with best response correspondence and Nash equilibria marked

	u	d
U	100*,100* (Efficient)	100,100*
D	1,1	101*,2* (Efficient)

Nash equilibria / Subgame Pefect equilibria:

(U,u): Efficient
(D,d): Efficient

d) Extensive form with subgame perfect choices marked with dashed lines



Subgame perfect equilibria:

(d, U):Not efficient
(d, D):Efficient

normal form with best response correspondence and Nash equilibria marked

	U	D
u	0*,0	100,100*
d	0*,0* (Not efficient)	101*,0* (Efficient)

Nash equilibria:

(d, U):Not efficient
(d, D):Efficient

Problem 2

1.

2. Demand fundtion: $P = 17 - (x_1 + x_2)$

   $MC_1 = 1$

   $MC_2 = 3$

   (a)

   $\pi_1 = Px_1 - x_1 = [17 - (x_1 + x_2)]x_1 - x_1$ Profit function for firm 1

   $\pi_2 = Px_2 - 3x_2 = [17 - (x_1 + x_2)]x_2 - 3x_2$ Profit function for firm 2

   (b)

   $\frac{\partial \pi_1}{\partial x_1} = 17 - 2x_1 - x_2 - 1 = 0 \Rightarrow x_1 = \frac{16 - x_2}{2}$ Firm 1 reaction function

   $\frac{\partial \pi_2}{\partial x_2} = 17 - x_1 - 2x_2 - 3 = 0 \Rightarrow x_2 = \frac{14 - x_1}{2}$ Firm 2 reaction function

   Combining both reaction functions:

   Nash Equilibrium: $x_1 = 6$, $x_2 = 4$, $P = 7$

   (c) $$\pi_1 = \begin{cases} 0 & \text{if } P_1 > P_2 \\ \left(\frac{17 - P_1}{2}\right)(P_1 - 1) & \text{if } P_1 = P_2 \\ (17 - P_1)(P_1 - 1) & \text{if } P1 < P2 \end{cases}$$

   $$\pi_2 = \begin{cases} 0 & \text{if } P_2 > P_1 \\ \left(\frac{17 - P_2}{2}\right)(P_2 - 3) & \text{if } P_2 = P_1 \\ (17 - P_2)(P_2 - 3) & \text{if } P_2 < P_1 \end{cases}$$

   d) Assume at $P=3$ firm 2 decides not to produce. Then, the Nash Equilibrium is: $P^*=3$ $X^*=14$

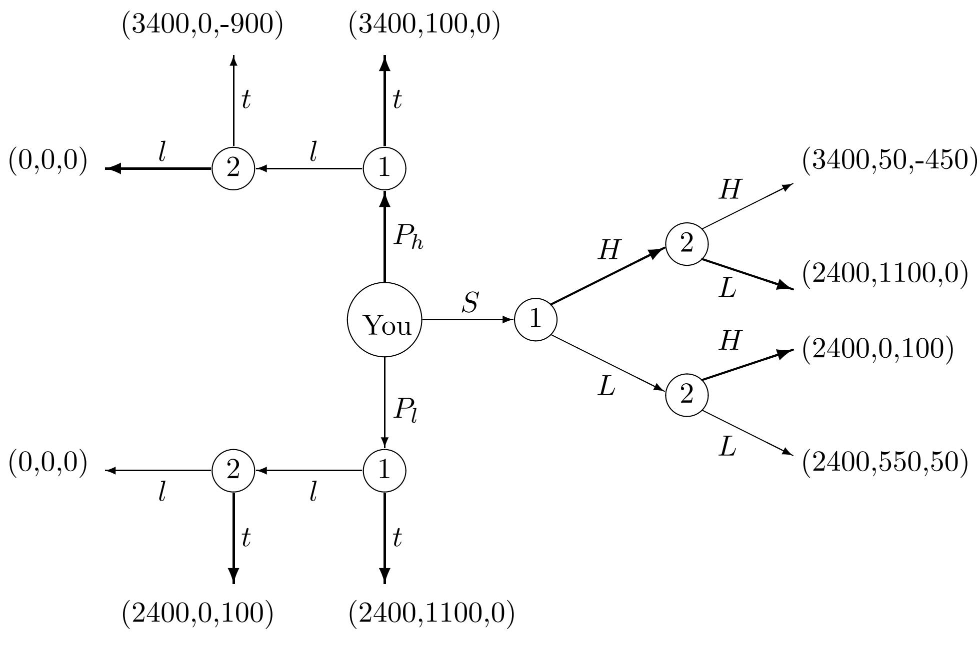
   e) More output is produced under Bertrand competition.

3. *How to sell a car:*

The three players in this game and the actions they can take are the following:

- You – you value the car at \$0
  - S: sell the car in a second price sealed bid auction
  - Ph: sell the car by setting a take-or-leave-it price of \$3400.
  - Pl: sell the car by setting a take-or-leave-it price of \$2400.
- Buyer 1 – values the car at \$3500
  - $H(L)$: bid \$3400 (\$2400) in the 2nd price auction
  - $t(l)$: take (leave) the set price in the take-it-or-leave-it scheme
- Buyer 2 – values the car at \$2500
  - $H(L)$: bid \$3400 (\$2400) in the 2nd price auction
  - $t(l)$: take (leave) the set price in the take-it-or-leave-it scheme

a) The extensive form. Note that the payoffs are the triples of (you,buyer1,buyer2).



b) The subgame perfect equilibrium of this three player game is

$$SPE: \quad (P_h, Htt, LHlt)$$

as illustrated by the thick lines in the extensive form.

- P_h is your strategy –
  P_h: set a take-it-or-leave-it price of \$3400.

- Htt is buyer 1's strategy –
  H: bid \$3400 if you sell the car in a second price auction; t: take if you set the take-or-leave-it price of \$3400; t: take if you set the take-or-leave-it price of \$2400.

- $LHlt$ is buyer 2's strategy –
  L: bid \$2400 if you sell the car in a second price auction and buyer 1 bid \$3400; H: bid \$3400 if you sell the car in a second price auction and buyer 1 bid \$2400; l: leave if you set the take-or-leave-it price of \$3400 (and buyer 1 leaves); t: take if you set the take-or-leave-it price of \$2400 (and buyer 1 leaves).