Review Answers From Economics 11

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1. Consumer and Demand Theory

income $1,000,000

utility is $\log x^1 + 2\log x^2$

marginal rate of substitution $\frac{\partial u/\partial x^1}{\partial u/\partial x^2} = \frac{x^2}{2x^1} = \frac{p^1}{p^2}$ or $p^2 x^2 = 2p^1 x^1$

budget constraint $p^1 x^1 + p^2 x^2 = I$

substitute and get $x^1 = \frac{I}{3p^1} = \frac{1{,}000{,}000}{3p^1}$

elasticity of demand for champagne $\frac{p^1}{x^1}\frac{\partial x^1}{\partial p^1} = -\frac{p^1}{x^1}\frac{I}{3}\frac{1}{(p^1)^2} = -1$

so a 10% price increase in champagne results in a 10% fall in demand for champagne

cross elasticity of demand for champagne $\frac{p^2}{x^1}\frac{\partial x^1}{\partial p^2} = \frac{p^2}{x^1}0 = 0$

so a 10% price increase in diamonds does not change the demand for champagne

2. General Equilibrium Theory

Rockstar

demand $x_R^1 = \frac{I}{3p^1}$; excess demand $z_R^1 = \frac{1000p^1 + 100p^2}{3p^1} - 1000$

Turkeyfeathers

demand $x_T^1 = \frac{2I}{3p^1}$; excess demand $z_T^1 = \frac{200p^1 + 40p^2}{3p^1} - 100$

aggregate excess demand

$$z_R^1 + z_T^1 = \frac{1000p^1 + 100p^2}{3p^1} - 1000 + \frac{200p^1 + 40p^2}{3p^1} - 100$$

$$= \frac{140p^2}{3p^1} - 700 = 0$$

solve for equilibrium price ratio $\frac{p^2}{p^1} = 15$

plug prices into demand to find equilibrium consumption

$$x_R^1 = \frac{1000}{3} + \frac{100}{3}15 = \frac{2500}{3}$$
$$x_T^1 = \frac{200}{3} + \frac{40}{3}15 = \frac{800}{3}$$

3. Lagrange Multipliers

Lagrangean is $\sqrt{x^1} + 2\sqrt{x^2} + 3\sqrt{x^3} - \lambda(p^1x^1 + p^2x^2 + p^3x^2 - I)$

$$\frac{\partial L}{\partial x^i} = i\frac{1}{2}\left(x^i\right)^{-1/2} - \lambda p^i = 0$$

solving we get $x^i = \dfrac{i^2}{4\lambda^2\left(p^i\right)^2}$

plugging into the budget constraint we get

$$\frac{1}{4\lambda^2 p^1} + \frac{4}{4\lambda^2 p^2} + \frac{9}{4\lambda^2 p^3} = I$$

or

$$\lambda = \sqrt{\frac{1}{4Ip^1} + \frac{4}{4Ip^2} + \frac{9}{4Ip^3}}$$

substituting back in we get the answer

$$x^1 = \frac{1}{4\left(\frac{1}{4Ip^1} + \frac{4}{4Ip^2} + \frac{9}{4Ip^3}\right)\left(p^1\right)^2}$$

as a check observe that this function is homogeneous of degree zero in prices and income:

if prices and income both double, demand (a real quantity) does not change.