Monopoly

Big Media Giant (BMG), the gigantic media company has a new album by the group Lucky in the Park.

What price should it charge for this new product?

Each unit will cost *c$* to produce and distribute.

Market research indicates that the number of units that will be sold x depends upon the price p according to the relation $x = d(p)$

Monopoly Solution

p is price, x is output, c is unit cost

profit $\pi = px - cx$

inverse demand $x = d(p)$ or $p = f(x)$

profit again $\pi = f(x)x - cx$

marginal profit equals zero

$$\frac{d\pi}{dx} = f'(x)x + f(x) - c = 0,\; f(x)\left[\frac{f'(x)x}{f(x)} + 1\right] = c$$

$$p\left[\frac{d\log p}{d\log x} + 1\right] = c$$

Discussion of the Solution

$$p\left[\frac{d\log p}{d\log x}+1\right]=c$$

$\dfrac{d\log p}{d\log x}$ is negative so $p > c$

- monopoly vs. “competition”
- the more “inelastic” is price with respect to output, the bigger the markup
- take into account how other “players” respond to your “strategy”: the more you sell, the lower the price “opponents” are willing to pay

An Example with Linear Demand

$$p = a - bx$$

monopoly

$$\pi = (a - bx)x - cx = (a - c)x - bx^2$$

$$\frac{d\pi}{dx} = (a - c) - 2bx = 0$$

$$x = \frac{a - c}{2b}$$

competitive equilibrium

$$p = c$$

$$a - bx = c$$

$$x = \frac{a - c}{b}$$

Graphical Analysis

revenue = $px = f(x)x$

marginal revenue = $MR = \frac{d}{dx}\text{revenue}$

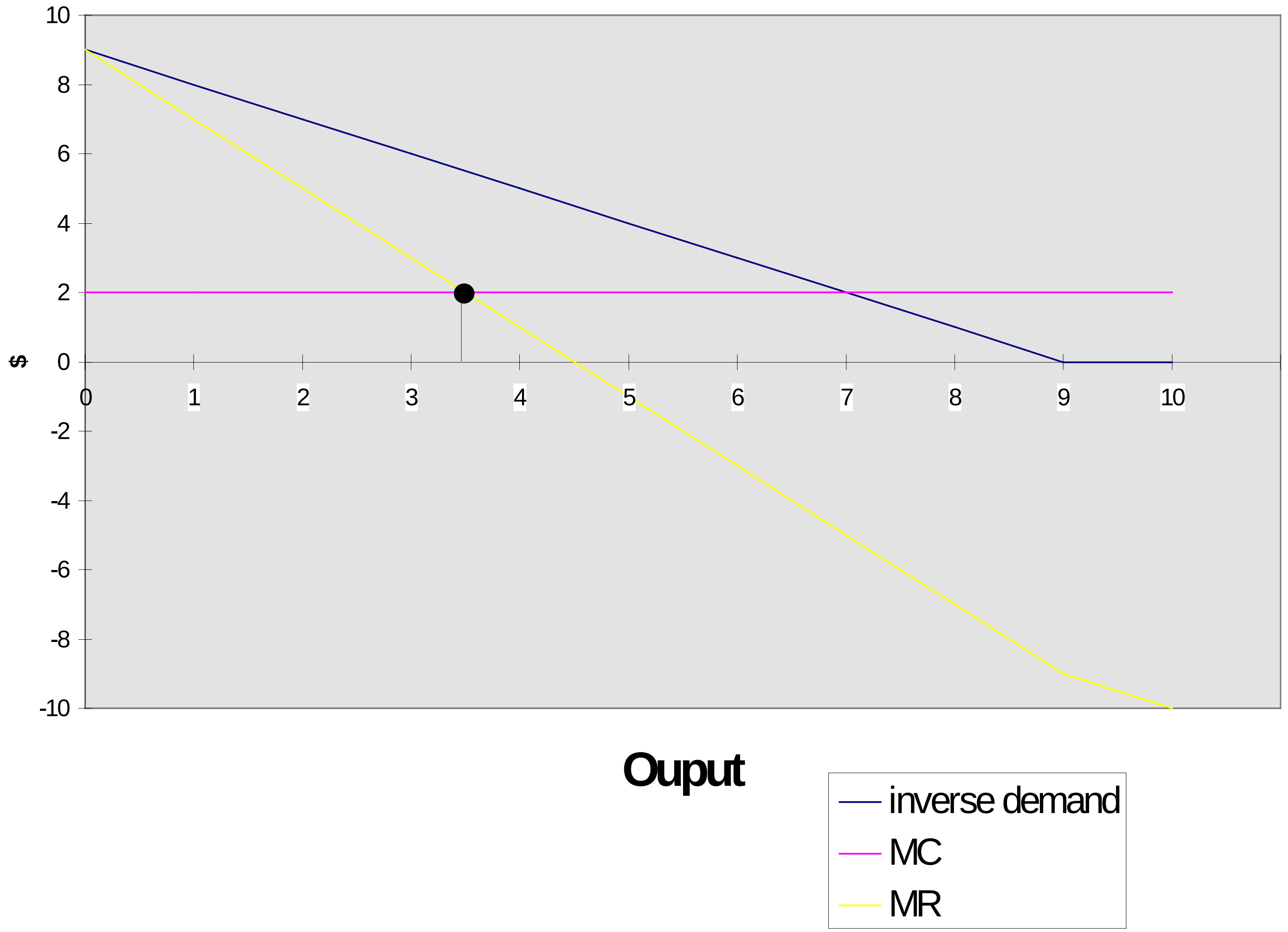
cost = cx

marginal cost= $MC = \frac{d}{dx}\text{cost} = c$

$f'(x)x + f(x) = c$ or $MR = MC$

take a=9, b=1, c=2

Optimum of the Monopolist



Returns to Scale

$$\text{total cost} = cx + dx^2 / 2$$

$$\text{average} = c + dx / 2$$

$$\text{marginal} = c + dx$$

- if $d = 0$ constant returns to scale
- if $d > 0$ decreasing returns to scale
- if $d < 0$ increasing returns to scale

Example Revisited

$$p = a - bx$$

monopoly

$$\pi = (a - bx)x - cx - dx^2 / 2$$

$$= (a - c)x - (b + d / 2)x^2$$

$$\frac{d\pi}{dx} = (a - c) - 2(b + d / 2)x = 0$$

$$x = \frac{a - c}{2b + d}$$

competitive equilibrium

$$a - bx = c + dx$$

$$x = \frac{a - c}{b + d}$$

- when $d > 0$ (decreasing returns to scale) monopolist produces more than ½ competition
- when $d < 0$ competitor earns negative profit

$$\text{average} = c + dx / 2$$

$$\text{marginal} = c + dx$$

when $d < 0$

average cost > marginal cost

so price = marginal cost < average cost

means you lose money on each unit you sell