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# Monopoly

Big Media Giant (BMG), the gigantic media company has a new album by the group Lucky in the Park.

What price should it charge for this new product?

Each unit will cost  $c$ \$ to produce and distribute.

Market research indicates that the number of units that will be sold  $x$  depends upon the price  $p$  according to the relation  $x = d(p)$

## ***Monopoly Solution***

$p$  is price,  $x$  is output,  $c$  is unit cost

$$\text{profit } \pi = px - cx$$

inverse demand  $x = d(p)$  or  $p = f(x)$

$$\text{profit again } \pi = f(x)x - cx$$

marginal profit equals zero

$$\frac{d\pi}{dx} = f'(x)x + f(x) - c = 0, \quad f(x) \left[ \frac{f'(x)x}{f(x)} + 1 \right] = c$$

$$p \left[ \frac{d \log p}{d \log x} + 1 \right] = c$$

## ***Discussion of the Solution***

$$p \left[ \frac{d \log p}{d \log x} + 1 \right] = c$$

$\frac{d \log p}{d \log x}$  is negative so  $p > c$

- monopoly vs. “competition”
- the more “inelastic” is price with respect to output, the bigger the markup
- take into account how other “players” respond to your “strategy”: the more you sell, the lower the price “opponents” are willing to pay

## *An Example with Linear Demand*

$$p = a - bx$$

monopoly

$$\pi = (a - bx)x - cx = (a - c)x - bx^2$$

$$\frac{d\pi}{dx} = (a - c) - 2bx = 0$$

$$x = \frac{a - c}{2b}$$

competitive equilibrium

$$p = c$$

$$a - bx = c$$

$$x = \frac{a - c}{b}$$

## ***Graphical Analysis***

$$\text{revenue} = px = f(x)x$$

$$\text{marginal revenue} = MR = \frac{d}{dx} \text{revenue}$$

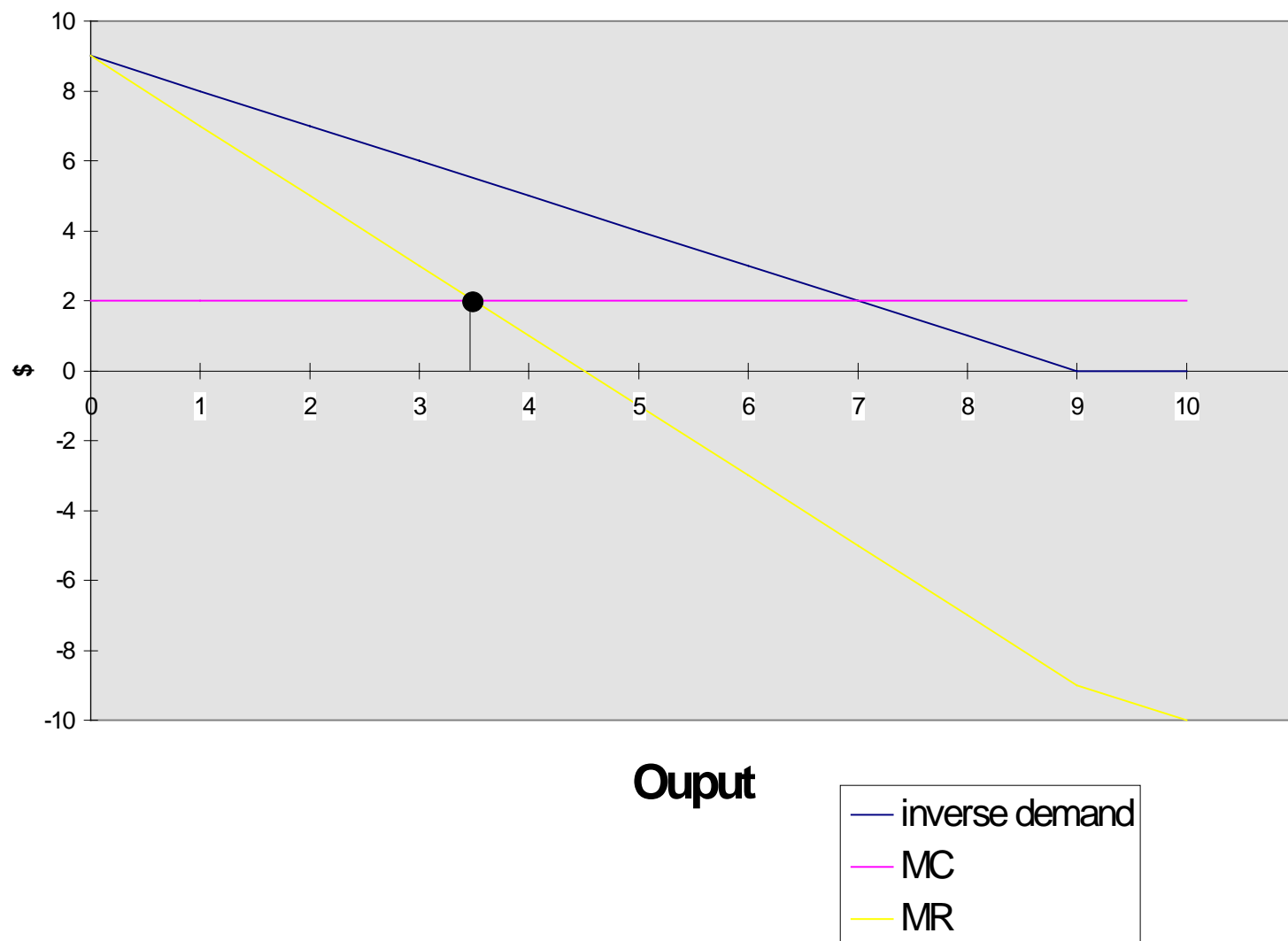
$$\text{cost} = cx$$

$$\text{marginal cost} = MC = \frac{d}{dx} \text{cost} = c$$

$$f'(x)x + f(x) = c \text{ or } MR = MC$$

take  $a=9$ ,  $b=1$ ,  $c=2$

## Optimum of the Monopolist



## Returns to Scale

$$\text{total cost} = cx + dx^2 / 2$$

$$\text{average} = c + dx / 2$$

$$\text{marginal} = c + dx$$

- if  $d = 0$  constant returns to scale
- if  $d > 0$  decreasing returns to scale
- if  $d < 0$  increasing returns to scale



## ***Example Revisited***

$$p = a - bx$$

monopoly

$$\pi = (a - bx)x - cx - dx^2 / 2$$

$$= (a - c)x - (b + d/2)x^2$$

$$\frac{d\pi}{dx} = (a - c) - 2(b + d/2)x = 0$$

$$x = \frac{a - c}{2b + d}$$

competitive equilibrium

$$a - bx = c + dx$$

$$x = \frac{a - c}{b + d}$$

- when  $d > 0$  (decreasing returns to scale) monopolist produces more than  $\frac{1}{2}$  competition
- when  $d < 0$  competitor earns negative profit

$$\text{average} = c + dx / 2$$

$$\text{marginal} = c + dx$$

when  $d < 0$

average cost > marginal cost

so price = marginal cost < average cost

means you lose money on each unit you sell