Auctions and Competition

Roth et al: 10 players submit bids (first price auction) on a prize worth $10

after a few rounds everyone is bidding $9.95

typical of games in a competitive environment

“Cournot” example with seven firms…competition or Cournot?

Dominance and The Prisoner's Dilemma Game

	cooperate	cheat
cooperate	2,2	0,3
cheat	3,0	1,1

- Has a unique dominant strategy equilibrium cheat-cheat
- This is Pareto dominated by cooperate-cooperate
- Role for altruism?

Public Goods Experiment

Players randomly matched in pairs

May donate or keep a token

The token has a fixed commonly known public value of 15

It has a randomly drawn private value uniform on 10-20

V=private gain/public gain

So if the private value is 20 and you donate you lose 5, the other player gets 15; V= -1/3

If the private value is 10 and you donate you get 5 the other player gets 15; V=+1/3

Data from Levine/Palfrey, experiments conducted with caltech undergraduates, based on Palfrey and Prisbey

Coordination Results

V	donating a token
0.3	100%
0.2	92%
0.1	100%
0	83%
-0.1	55%
-0.2	13%
-0.3	20%

Weak Dominance and the Second Price Auction

- bidding your value is weakly dominant
- BDM mechanism with random “second highest bid”
- The endowment effect

This ticket is worth $2.00 to you.

You can sell it.

Name your offer price.

A price will be posted shortly

The posted price was drawn randomly between:

[$ 0 and $ 6]

If your offer price is **below** the posted price then you sell your ticket at the posted price.

If your offer price is **above** the posted price then you do not sell your ticket but you do collect the $2.00 value of the ticket.

You can view the posted price after you have named your price.

Indicate the appropriate amount .

My offer price is **below** the posted price.

Pay me the posted price of $__________.

My offer price is **above** the posted price.

Pay me $ 2.00.

Coordination Games

	L	R
U	2,2	0,0
D	0,0	1,1

three equilibria (U,L) (D,R) plus mixed

too many equilibria?? introspection possible?

the rush hour traffic game – introspection clearly impossible, yet we seem to observe Nash equilibrium

equilibrium through learning?

Coordinate on efficient equilibrium?

Coordination Experiments

Van Huyck, Battalio and Beil [1990]

Actions $A = \{1,2,\ldots 7\}$

Utility $u(a_i, a_{-i}) = b_0 \min(a_j) - ba_i$ where $b_0 > b > 0$

14-16 players

Everyone doing a' the same thing is always a Nash equilibrium

$a' = \overline{e}$ is efficient, the bigger is a' the more efficient, but the “riskier”

a model of “riskier” some probability of one player playing $a' = 1$

story of the stag-hunt game

Coordination Results

treatments: A $b_0 = 2b$, B $b = 0$

- In final period treatment A:

  77 subjects playing $a_i = 1$

  30 subjects playing something else

  minimum was always 1

- In final period treatment B:

  87 subjects playing $a_i = 7$

  0 playing something else

- with two players $a_i = 7$ was more common

Approximate Equilibria and Near Equilibria

- exact: $u_i(s_i|\sigma_{-i}) \geq u_i(s_i'|\sigma_{-i})$

  approximate: $u_i(s_i|\sigma_{-i}) + \varepsilon \geq u_i(s_i'|\sigma_{-i})$

- Approximate equilibrium can be very different from exact equilibrium

Radner's work on finite repeated PD

gang of four on reputation

upper and lower hemi-continuity

A small portion of the population playing "non-optimally" may significantly change the incentives for other players causing a large shift in equilibrium behavior.

Quantal Response Equilibrium

(McKelvey and Palfrey)

propensity to play a strategy

$p_i(s_i) = \exp(\lambda_i u_i(s_i, \sigma_{-i}))$

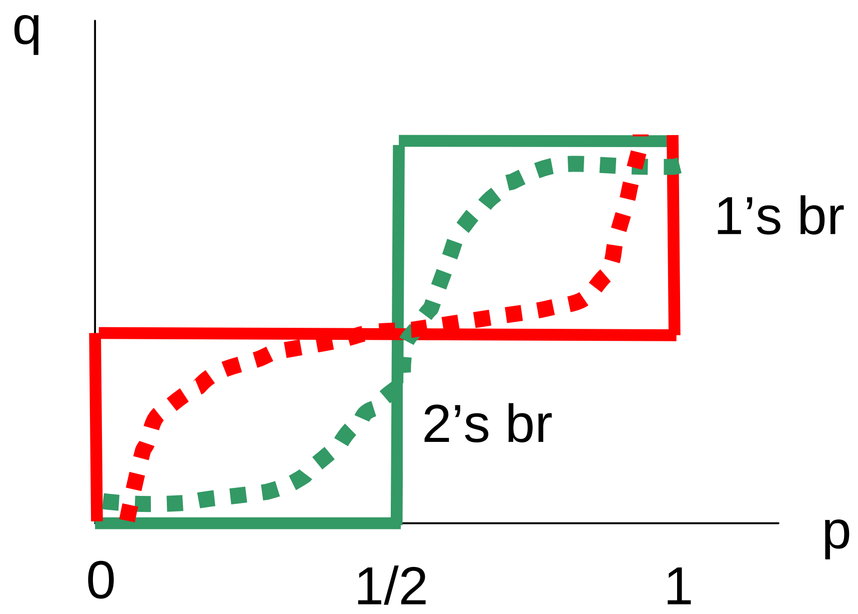
$\sigma_i(s_i) = p_i(s_i) / \sum_{s_i'} p_i(s_i')$

as $\lambda_i \to \infty$ approaches best response

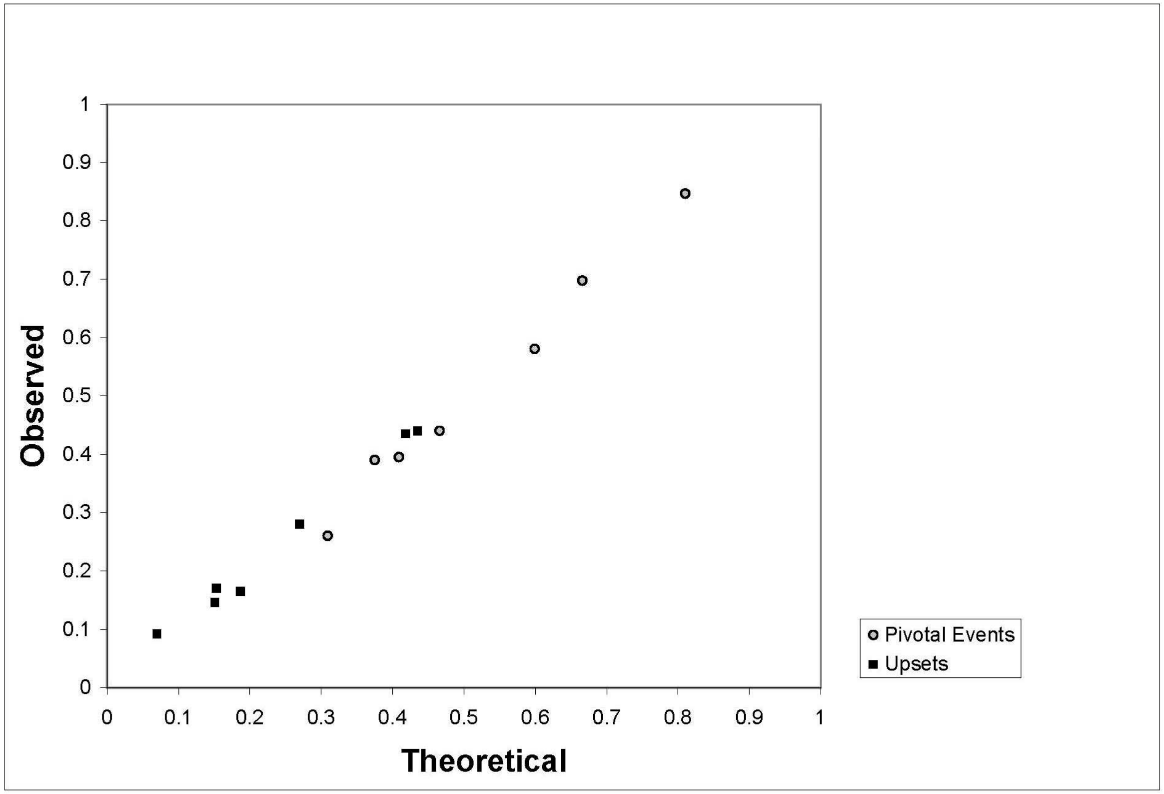
as $\lambda_i \to 0$ approaches uniform distribution

Smoothed Best Response Correspondence Example

	L ($\sigma_2(L) = q$)	R
U ($\sigma_1(U) = p$)	1,1	0,0
D	0,0	1,1

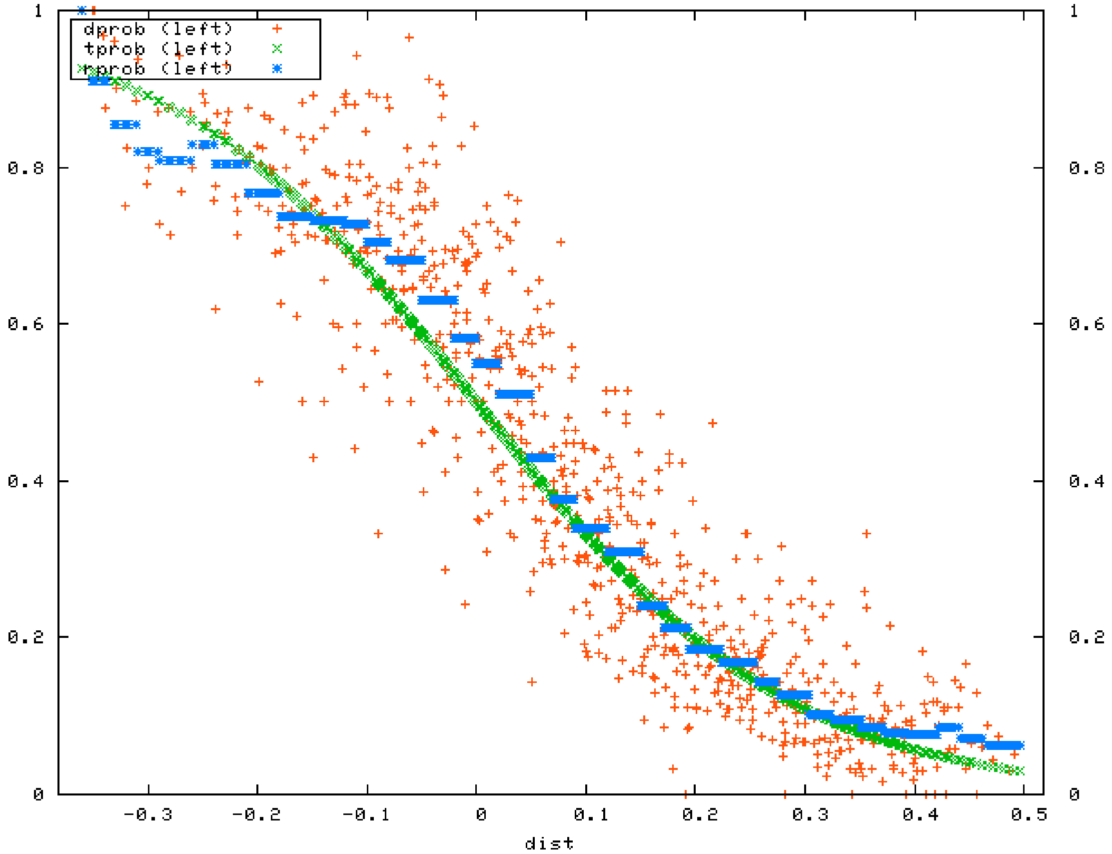


Voting



Observed
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Theoretical
Pivotal Events
Upsets

Individual Behavior



Observations

- contains an unknown preference parameter λ
- $\lambda = 0$ play is completely random
- as λ becomes large, the probability of playing the “best” response approaches one
- λ kind of index of rationality.
- in the voting experiment we can estimate a common value of λ for all players.
- corresponding equilibrium probabilities of play are given by the green curve
- does an excellent job of describing individual play
- it makes roughly the same predictions for aggregate play as Nash equilibrium

Limitations of QRE

- captures only the cost side of preferences
- recognizes – correctly – departures from standard “fully rational” selfish play are more likely if less costly in objective terms
- does not attempt to capture benefits of playing non-selfishly
- does not well capture, for example, the fact that under some circumstances players are altruistic, and in others spiteful.

Auctioning a Jar of Pennies

- surefire way to make some money
- put a bunch of pennies in a jar
- get together a group of friends
- auction off the jar of pennies
- with about thirty friends that you can sell a $3.00 jar of pennies for about $10.00

Winner's Curse

- friends all stare at the jar and try to guess how many pennies there are.
- Some under guess – they may guess that there are only 100 or 200 pennies. They bid low.
- Others over guess – they may guess that there are 1,000 pennies or more. They bid high.
- Of course those who overestimate the number of pennies by the most bid the highest – so you make out like a bandit.

Nash Equilibrium?

- According to Nash equilibrium this shouldn't happen
- Everyone should rationally realize that they will only win if they guess high
- they should bid less than their estimate of how many pennies there are in the jar
- they should bid a lot less – every player can guarantee they lose nothing by bidding nothing.
- in equilibrium, they can't on average lose anything, let alone $7.00.

QRE

- Recognize that there is small probability people aren’t so rational
- Very different prediction
- some most possible profit anyone can make by getting the most number of pennies at zero cost: call this amount of utility U
- some least possible profit by getting a jar with no pennies at the highest possible bid: call this amount of utility u
- QRE says ratio of probability between two bids that give utility U, u is $\exp[\lambda(U-u)]$
- whatever is the difference in utility between two strategies it cannot be greater than that between U and u
- probability of highest possible bid is at least $p > 0$
- depends on how many bids are possible, not on how many bidders or their strategies

QRE with Many Bidders

- each bidder has at least a p probability of making the highest possible bid
- becomes a virtual certainty that one of the bidders will (unluckily for them) make this high bid

with enough bidders, QRE assures the seller a nice profit.

Mixed Strategies: How Do Athletes Do It?

- Holmes, Moriarity, Canterbury and Dover
- once in Japan catchers equipped with mechanical randomization devices to call the pitch
- later ruled unsporting and banned from play
- good tennis players in important matches do it right
- professional soccer players do it right
- submarine captains and the RAND corporation

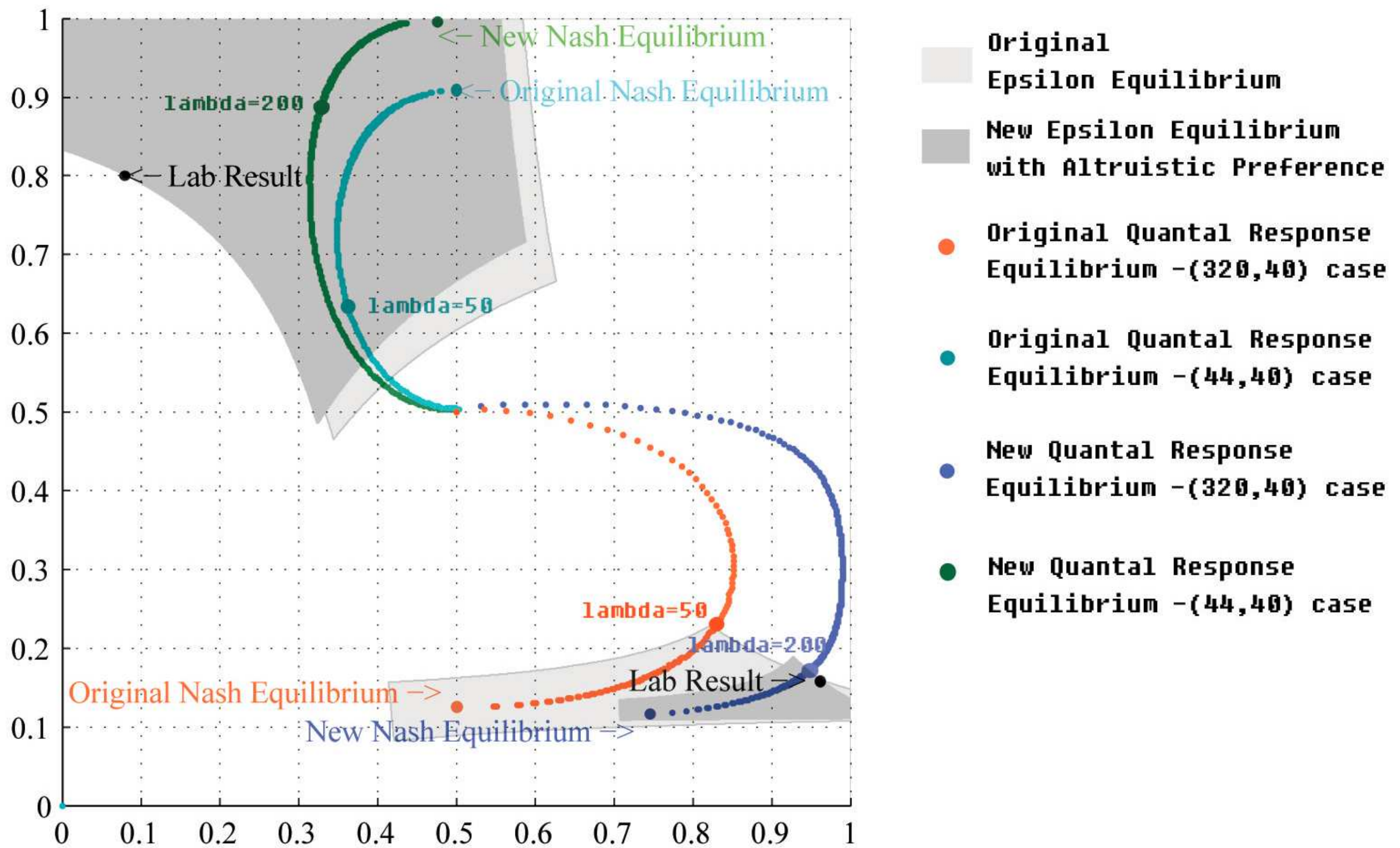
Goeree and Holt: Matching Pennies

Symmetric

	50% (48%)	50% (52%)
50% (48%)	80,40	40,80
50% (52%)	40,80	80,40

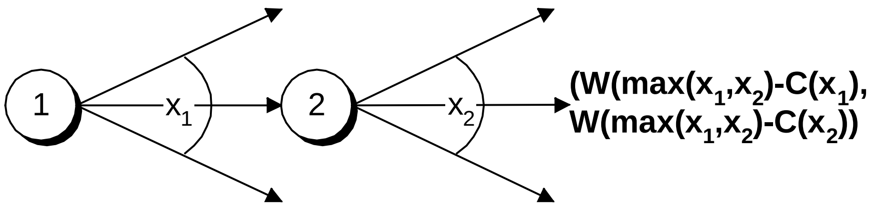
	12.5% (16%)	87.5% (84%)
50% (96%)	320,40	40,80
50% (4%)	40,80	80,40

	(80%)	(20%)
50% (8%)	44,40	40,80
50% (92%)	40,80	80,40


Original
Epsilon Equilibrium
New Epsilon Equilibrium
with Altruistic Preference
Original Quantal Response
Equilibrium -(320,40) case
Original Quantal Response
Equilibrium -(44,40) case
New Quantal Response
Equilibrium -(320,40) case
New Quantal Response
Equilibrium -(44,40) case
<- New Nash Equilibrium
<- Original Nash Equilibrium
lambda=200
<- Lab Result
lambda=50
lambda=50
lambda=200
Lab Result ->
Original Nash Equilibrium ->
New Nash Equilibrium ->
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Subgame Perfection and Best Shot

Prasnikar and Roth



x	*W(x)*	*C(x)*
0	$0.00	$0.00
1	$1.00	$0.82
2	$1.95	$1.64
3	$2.85	$2.46
4	$3.70	$3.28
5	$4.50	$4.10
6	$5.25	$4.92
7	$5.95	$5.74
8	$6.60	$6.50

Discussion of Best Shot

if the other player makes any contribution at all, it is optimal to contribute nothing

unique subgame perfect equilibrium player 1 contributes nothing

another Nash equilibrium player 2 to contributes nothing regardless of player 1’s play

Best-Shot Results

Hirshleifer-Harrison partial information, but alternating roles

Prasnikar-Roth fixed roles, both partial and full information

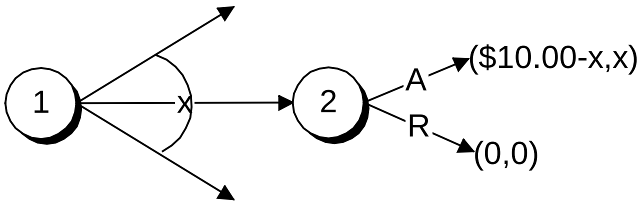
In the full information case and partial information heterogeneous case player 2 occasionally contributes less than 4 when player 1 has contributed nothing; Note that the player who contributes nothing gets \$3.70 against \$0.42 for the opponent who contributes 4

- full information case: player 1 never contributed anything
- partial information case: sometimes roles reverse

Subgame Perfection and Ultimatum Bargaining

player 1 proposes how to divide $10 in nickles

player 2 may accept or reject



Nash: any proposal by player 1 with all poorer proposals rejected and equal or better proposals accepted

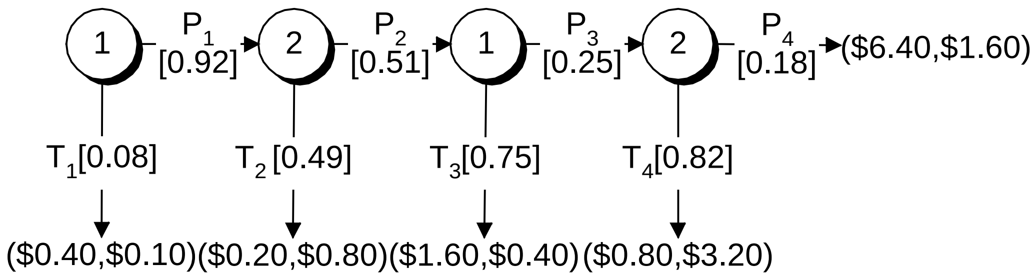
Subgame Perfect: First player gets at least $9.95

US Data for Ultimatum

x	*Offers*	*Rejection Probability*
\$2.60	3	33%
\$4.25	13	18%
\$5.00	13	0%
	29	

US \$10.00 stake games, round 10

Centipede Game: Palfrey and McKelvey



Numbers in square brackets correspond to the observed conditional probabilities of play corresponding to rounds 6-10, stakes 1x below.

This game has a unique Nash equilibrium path; in it player 1 with probability 1 plays T_1